THE INSTITUTE OF EDUCATION

Maths

Jean Kelly

Ordinary Level

2020-21

Complex Numbers

MATHS (O) NOTES

SUBJECT: Maths
LEVEL: Ordinary
TEACHER: Jean Kelly

Topics Covered:
- Complex Numbers

About Jean:

Jean has a wide breadth of experience in teaching Leaving Cert Ordinary Level Maths to students of all abilities and has been teaching in The Institute of Education for over 10 years. Over that time, Jean has developed an unmatched track record in helping students through the Maths syllabus and brings a refreshing approach to the explanation, clarification and tuition of the Maths syllabus.

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COMPLEX NUMBERS
Strand 3(Unit 1)

Syllabus

- Understanding the origin and need for complex numbers and how they are used to model 2D systems: as in computer games, alternating current and voltage.
- How to interpret multiplication by i as a rotation of $90°$ anticlockwise.
- How to express complex numbers in the rectangular form ($a+bi$) and to illustrate complex numbers on an Argand diagram.
- How to investigate the operations of addition and subtraction of complex numbers using the Argand diagram.
- How to investigate the operations of addition, subtraction, multiplication and division with complex numbers C in the form $a+bi$ (rectangular form) and calculate the complex conjugate as a reflection in the real axis.
- How to interpret the Modulus as distance from the origin on an Argand diagram.
- How to interpret multiplication by a complex number as a "multiplication of" the modulus by a real number combined with a rotation.
- How to solve Quadratic Equations having complex roots and how to interpret the solutions.

Imaginary numbers

There exists no real numbers that, when squared, result in a negative number:

$$x^2+1=0$$
$$x^2=-1$$
$$x=\sqrt{-1}$$

$$\boxed{\sqrt{-1}\notin R}$$

To overcome this difficulty an ***"imaginary number"*** **" i "** was introduced, where $i^2=-1$

This allows for the square root of negative numbers to be found:

$$\sqrt{-x}=\sqrt{x}\sqrt{-1}=\sqrt{x}(i)$$

Imaginary numbers take the form bi , where $b\in R,\ i=\sqrt{-1}.$

Complex numbers

Typically the variable used for real numbers is *x*. For complex numbers we often use the variable *z*, where $z = a + bi$,
where $a,b \in R,\ i^2 = -1\ and\ i = \sqrt{-1}$.

A complex number is written in this way:

1. *a* is called the **real part**, and is written as Re
2. *bi* is called the **imaginary part**, and is written as Im

The set of all complex numbers is C ,

$$C = \{a + bi \mid a,b \in R,\ i^2 = -1\}$$

Addition & Subtraction of Complex numbers

When adding or subtracting complex numbers, add or subtract the real parts, then add or subtract the imaginary parts.

$$(a+bi)+(c+di)=(a+c)+(b+d)i$$

And

$$(a+bi)-(c+di)=(a-c)+(b-d)i$$

$z = 2+3i$ and $w = 1-5i$

Q1) Calculate $z + w$

$$= (2+3i)+(1-5i)$$
$$= 2+1+3i-5i$$
$$= 3-2i$$

Q2) Calculate $z - w$

$$= (2+3i)-(1-5i)$$
$$= 2+3i-1+5i$$
$$= 2-1+3i+5i$$
$$= 1+8i$$

Multiplication of Complex numbers

Use the same approach as you'd use when multiplying polynomials together in algebra; multiply every term in one bracket by every term in the other.

$$(a+bi)(c+di) = ac + adi + bci + bdi^2$$
$$= ac + (ad+bc)i + bd(-1)$$
$$= (ac-bd) + (ad+bc)i$$

$z = 2+3i \quad and \quad w = 1-5i$

Q2) Calculate zw

$= (2+3i)(1-5i)$ $\quad \boxed{i^2 = -1}$

$= 2 - 10i + 3i - 15i^2$

$= 2 - 7i - 15(-1)$

$= 2 - 7i + 15$

$= 17 - 7i$

Complex Conjugate

In order to divide complex numbers we must first define the conjugate of a complex number.
If $z = a + bi$, then the complex conjugate of z, written as $\overline{z}$, is defined by:

$$\overline{z} = a - bi$$

To find the complex conjugate just change the ***sign* of the *imaginary part***.

The conjugate of a complex number is a reflection/ **image through the *x-axis* (y-value of point changes sign)**

$$z.\overline{z} = (a+bi)(a-bi)$$
$$= aa + (ab-ba)i - bb(-1)$$
$$= a^2 + b^2$$

Q. Calculate $z.\overline{z}$, where $z = 2+3i$.

$= (2+3i)(2-3i)$

$= 4 - \cancel{6i} + \cancel{6i} - 9i^2$

$= 4 - 9(-1)$

$= 4 + 9$

$= 13$ $\quad \boxed{real\ number}$

If a complex number is **added** to or **multiplied by its conjugate**, the **answer** will always be a **real** number.

If complex **conjugates** are **subtracted**, the **answer** is always an **imaginary** number.

Division of Complex numbers

As we cannot divide by a complex number, we must first multiply the fraction 'above & below' by the conjugate of the denominator.

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$$
$$= \frac{(a+bi)(c-di)}{c^2+d^2}$$
$$= \frac{(ac+bd)+(ad+bc)i}{c^2+d^2}$$
$$= \frac{ac+bd}{c^2+d^2} + \frac{ad+bc}{c^2+d^2}i$$

Q. Express $\frac{z}{w}$ in the form $a+bi$, where $a,b \in R,\ i^2=-1$

$$= \frac{2+3i}{1-5i} \times \left(\frac{1+5i}{1+5i}\right) \qquad [\times \textit{ by conjugate of bottom}]$$
$$= \frac{2+10i+3i+15i^2}{1+\cancel{5i}-\cancel{5i}-25i^2}$$
$$= \frac{2+13i+15(-1)}{1-25(-1)}$$
$$= \frac{2+13i-15}{1+25}$$
$$= \frac{-13+13i}{26} \qquad [\div \textit{ top and bottom by } 13]$$
$$= -\frac{1}{2}+\frac{1}{2}i$$

Example ①. Basic Operations with Complex numbers

Q1 Simplify and write your answer in the form $a+bi$:

(i) $(2+3i)+(4-5i)$ **(ii)** $(-4+i)-(3+2i)$

(iii) $2(5+2i)-(6-3i)$ **(iv)** $(1+3i)^2+2(-2+5i)$

(v) $(3+4i)(5-6i)$ **(vi)** $(8-3i)-2i(7+4i)$

(vii) $(4+2i)(3-i)$ **(viii)** $2(3-5i)+7i(2+3i)$

(ix) $3(2-4i)+i(5-6i)$ **(x)** $4(2-i)+i(3+5i)$

Q2 Simplify and write your answer in the form $a+bi$:

(i) $2(3-i)+i(4+5i)$ **(ii)** $7(2+i)+i(11+9i)$

(iii) $3(1+5i)+i(3-2i)$ **(iv)** $4i(2-3i)+7(-2-4i)$

(v) $3(4+i)+i(2-5i)$ **(vi)** $4i+i(3-2i)-1$

(vii) $(7+2i)+(5+6i)$ **(viii)** $(11+3i)-(5-2i)$

(ix) $(3+i)(-2-5i)$ **(x)** $(1+3i)i+2(3-i)^2$

kz

Example ②. Basic Operations with Complex numbers

$z=a+bi$
$=(a,b)$

Q1 Express in the form $a+bi$:

(i) $\frac{1}{1-i}$ **(ii)** $\frac{2i}{3+i}$ **(iii)** $\frac{2+3i}{3+4i}$ **(iv)** $\frac{4+2i}{3-i}$ **(v)** $\frac{6-8i}{4+3i}$

(vi) $\frac{4+2i}{1+2i}$ **(vii)** $\frac{17}{3+5i}$ **(viii)** $\frac{13}{3+2i}$ **(ix)** $\frac{3-2i}{1-4i}$ **(x)** $\frac{2}{1+3i}$

Q2 Express in the form $a+bi$:

(i) $\frac{5+12i}{2-3i}$ **(ii)** $\frac{6}{1+i}$ **(iii)** $\frac{5+4i}{5-4i}$ **(iv)** $\frac{1}{3-i+6i}$ **(v)** $\frac{3-6i}{3-6i+3i}$

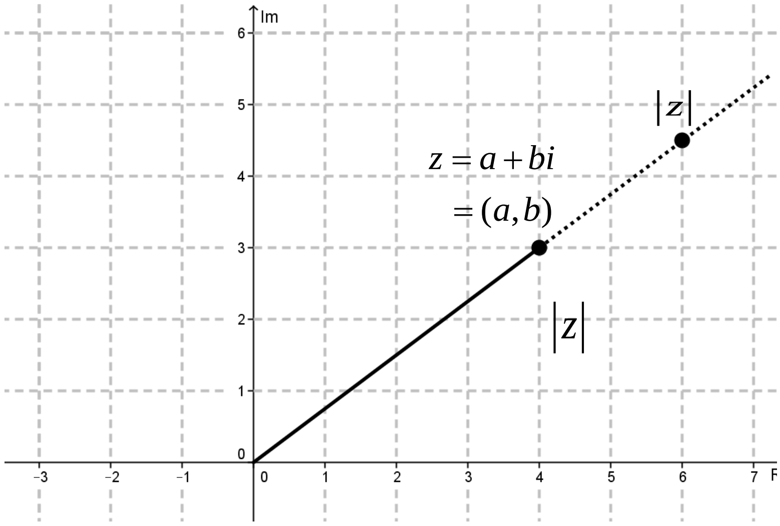
(vi) $2-i+\frac{1}{2-i}$ **(vii)** $\frac{1+i}{1-i}$ **(viii)** $\frac{2-5i}{2i}$ **(ix)** $\frac{i(3-4i)}{1+2i}$ **(x)** $\frac{5}{i^2(2-i)}$

Argand Diagram

We represent all real numbers on a **one-dimenional** number line.

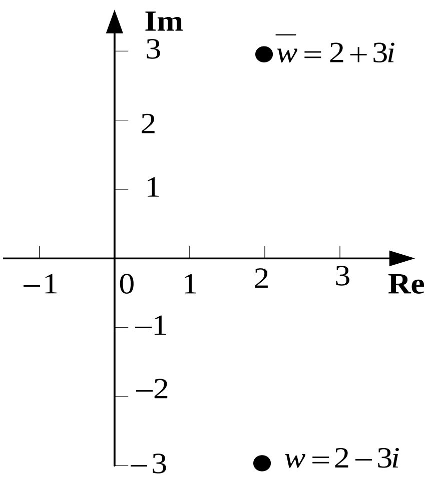
We use **two-dimensional plane** to represent complex numbers.

We use the **horizontal axis** to plot the **real part (Re)** and a **vertical axes** to plot the **imaginary part (Im).**



Plot complex numbers exactly like you plot points, where the x **co-ordinate** is the **real** part **(Re)** and the y **co-ordinate** is the **imaginary** part **(Im)**.

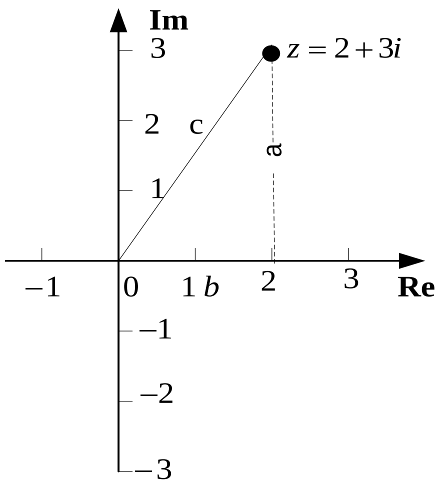
Q $w = 2 - 3i$. Plot w and $\overline{w}$ on an Argand diagram.



Any **multiple of *z*** is ***collinear*** with ***z* and the origin**

Modulus

The modulus of a complex number, $z = a + bi$, is the **distance from the origin** on an Argand diagram to the point (a, b).



The modulus of *z* is written as $|z|$, where $|z| = \sqrt{a^2 + b^2}$

This formula comes from Pythagoras theorem: $c^2 = a^2 + b^2$

$$z = 2 + 3i, \ w = 1 - 5i$$

Q1) Calculate $|z|$

$$= |2 + 3i| = \sqrt{(2)^2 + (3)^2} = \sqrt{4 + 9} = \sqrt{13}$$

Q2) Calculate $|\bar{z}|$

$$= |2 - 3i| = \sqrt{(2)^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$$

Q3) Calculate $|w|$

$$= |1 - 5i| = \sqrt{(1)^2 + (-5)^2} = \sqrt{1 + 25} = \sqrt{26}$$

$z = 2+3i,\ w = 1-5i$

$Q.1\ Calculate\ |z|$

$= |2+3i| = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}$

$Q.2\ Calculate\ |\overline{z}|$

$= |2-3i| = \sqrt{(2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$

$Q.3\ Calculate\ |w|$

$= |1-5i| = \sqrt{(1)^2+(-5)^2} = \sqrt{1+25} = \sqrt{26}$

Example ③: Argand Diagrams & Modulus

Q1 If $z_1 = 2-3i$ and $z_2 = -3+i$, plot the following complex numbers on an Argand diagram:

(i) z_1 **(ii)** z_2

(iii) $2z_1 + 3z_2$ **(iv)** $z_1 z_2$

Q2 Let $u = 1+2i,\ where\ i^2 = -1.$ Plot on an Argand diagram:

(i) u **(ii)** $u-3.$

Q3 Let $u = 3-4i,\ where\ i^2 = -1.$ Plot on an Argand diagram:

(i) u **(ii)** $u+5i.$

Q4 Let $z = 5-3i.$ Plot $z\ and\ -z$ on an Argand diagram.

Q5 Let $u = 4-2i,\ where\ i^2 = -1.$ Plot on an Argand diagram:

(i) u **(ii)** $u-4.$

Q6 Let $w = 1-2i.$ Plot $w\ and\ \overline{w}$ on an Argand diagram.

Q7 Let $z_1 = 2+3i\ and\ z_2 = 5-i.$ Plot $z_1,\ z_2\ and\ z_1+z_2$ on an Argand diagram.

Q8 Let $w = 3-2i,\ where\ i^2 = -1.$ Plot on an Argand diagram:

(i) w **(ii)** $iw.$

Q9 $z = 1+i,\ where\ i^2 = -1.$

(i) Plot $z,\ z^2,\ z^3\ and\ z^4$ on an Argand diagram.

(ii) Make one observation about the pattern of the points on the diagram.

Q10 If $z = 4+2i$ calculate $|z^2 - 4z|$.

Q11 If $z = 3-2i$ calculate $|z^2 - 4\overline{z} + 4 + i|$.

Q12 If $z = 2+5i$ and $w = -1+2i$, investigate if $|z+w| = |z| + |w|$.

Q13 $z = 8+ki$, where $k \in R$. If $|z| = 10$, find the possible values of *k*.

Q14 $z = 3+5i$. If $|z+ki| = \sqrt{58}$, find the possible values of $k \in R$.

Q15 $u = 4+3i\ and\ w = 6-8i.$ Find the value of the real number k such that $|u| = k|w|$

2012 Ordinary Level Paper 1: Q3 (25 Marks)

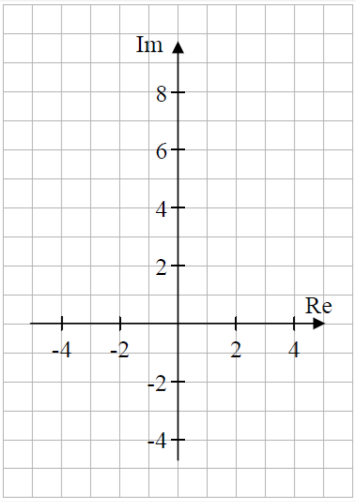
The complex number $z = 1 - 4i$, where $i^2 = -1$.

(a) Plot *z* and $-2z$ on the Argand diagram.

(b) Show that $2|z| = |-2z|$.

(c) What does part **(b)** tell you about the points you plotted in part **(a)**?

(d) Let *k* be a real number such that $|z + k| = 5$. Find the two possible values of *k*.



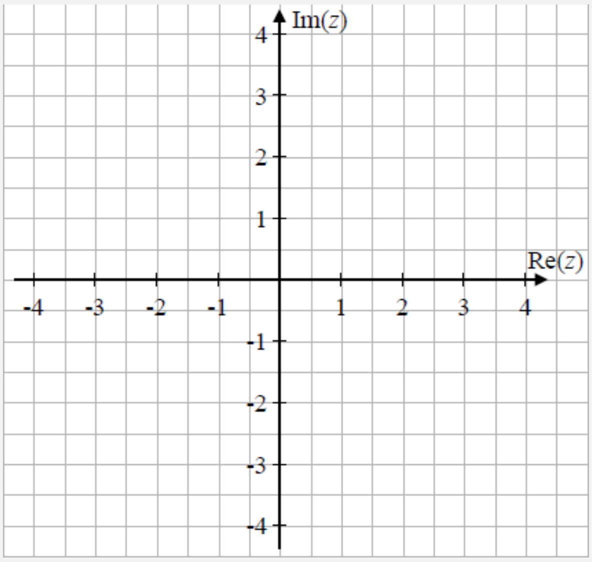
2011 SEC Ordinary Level Sample P1: Q3 (25 Marks)

Two complex numbers are
$u = 3 + 2i$ and $v = -1 + i$,
where $i^2 = -1$.

(a) Given that $w = u - v - 2$, evaluate w.

(b) Plot *u*, *v*, and *w* on the Argand diagram given.

(c) Find $\frac{2u + v}{w}$.



Solving/Finding Roots of Quadratic Equations

Use the Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (pg. 20) log tables

The **two roots/solutions** are always **conjugates**

Q Solve $z^2 + 6z + 13 = 0$ and write your answers in the form $a \pm bi,\ where\ a, b \in R.$

$$z^2 + 6z + 13 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad sub\ in:\ a = 1, b = 6\ \&\ c = 13$$

$$= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 52}}{2}$$

$$= \frac{-6 \pm \sqrt{-16}}{2}$$

$$= \frac{-6 \pm 4i}{2} \qquad .....\sqrt{-16} = \sqrt{16} \times \sqrt{-1} = 4 \times i = 4i$$

$$= -3 \pm 2i \qquad (\text{Conjugates})$$

N.B. If you **solve a quadratic equation** $(Let\ y = 0)$ to find the roots/solutions/ $x - values$, you are trying to find where the "Happy/Sad face" curve **cuts the** $x - axis$. If the **solutions are complex numbers**, then the **curve does not touch the x-axis!** Try and draw a graph of the function $x^2 + 6x + 13 = 0$, where $-6 \le x \le 0$

N.B. When trying to find the **points of intersection** between a **line** and a **curve/circle** in algebra, you may end up with a **quadratic equation** that results in **solutions** that are **complex numbers** and therefore the **line does not intersect the curve or circle!** Try and solve the equations $x - y + 6 = 0$ and $x^2 + y^2 = 10$ for the points of intersection

Verifying Roots of Quadratic Equations

To prove that a **complex number is a root** of a quadratic equation, **substitute** the complex number into the equation for the variable and the **answer should be equal to zero.**

Q1. Verify that $2+3i$ is a root of the equation $z^2-4z+13=0$ and write down the other root.

Let $z=2+3i$, Sub in:

$z^2-4z+13=0$

$(2+3i)^2-4(2+3i)+13=0$

$(2+3i)(2+3i)-4(2+3i)+13=0$

$4+6i+6i+9i^2-8-12i+13=0$

$4+12i+9(-1)-8-12i+13=0$

$4-9-8+13=0$

$-5+5=0$

$0=0$

$\boxed{Other\ root=conjugate}$

$\boxed{\overline{z}=2-3i}$

Q2. $z=-4+i$ is one root of the equation $z^2+8z+k=0$, find the value of k and write down the other root.

Let $z=-4+i$, Sub in:

$z^2+8z+k=0$

$(-4+i)^2+8(-4+i)+k=0$

$(-4+i)(-4+i)+8(-4+i)+k=0$

$16-4i-4i+i^2-32+8i+k=0$

$16-8i+(-1)-32+8i+k=0$

$16-1-32+k=0$

$-17+k=0$

$k=17$

$\boxed{Other\ root=conjugate}$

$\boxed{\overline{z}=-4-i}$

If you **know the roots** of a quadratic equation, use the formula:

$$z^2 - z(sum\ of\ roots) + product\ of\ roots = 0$$

to **form the Quadratic** Equation.

Q3. If $z = 4+5i$ is a root of the equation $z^2 + bz + c = 0$. Find the value of b and the value of c.

$z = 4+5i$ *and* $\overline{z} = 4-5i$ *are the two roots*

Sub in:

$z^2 - z(sum\ of\ roots) + product\ of\ roots = 0$

$z^2 - z((4+5i)+(4-5i)) + (4+5i)(4-5i) = 0$

$z^2 - z(4+4\cancel{+5i}\cancel{-5i}) + 16\cancel{-20i}\cancel{+20i} - 25i^2 = 0$

$z^2 - z(8) + 16 - 25(-1) = 0$

$z^2 - 8z + 41 = 0$ $\boxed{b=-8,\ c=41}$

Example 4. Solving Quadratic Equations and Verifying roots

Q1 Solve each equation and write the answers in the form $a \pm bi$.

(i) $z^2 - 4z + 20 = 0$ **(ii)** $z^2 - 10z + 26 = 0$ **(iii)** $z^2 - 4z + 29 = 0$

(iv) $z^2 - 6z + 34 = 0$ **(v)** $z^2 - 10z + 29 = 0$

Q2 Verify that each complex number is a root of the equation and write down the other root:

(i) $4+3i$, $z^2 - 8z + 25 = 0$ **(ii)** $-1+2i$, $z^2 + 2z + 5 = 0$

(iii) $5-4i$, $z^2 - 10z + 41 = 0$ **(iv)** $-7+i$, $z^2 + 14z + 50 = 0$

(v) $-6-i$, $z^2 - 12z + 37 = 0$

Q3 Form a quadratic equation with the roots $5 \pm 3i$.

Q4 If $-3+3i$ is one of the roots of the equation $z^2 + az + b = 0$, find the value of a *and* b.

Geometrical properties of Complex numbers (Transformations)

1. Rotations

A ***rotation*** turns a point through an angle about a fixed point

If a point (complex number) is **multiplied by** i, the number is rotated by $90°$ **anti-clockwise about the** *origin*. This is a *positive* rotation.

If a point (complex number) is **multiplied by** $-i$, the number is rotated by $90°$ **clockwise about the** *origin*. This is a *negative* rotation

On an Argand diagram:

Multiplication by i rotates a complex number by $90°$ anti-clockwise
Multiplication by i^2 rotates a complex number by $180°$ anti-clockwise
Multiplication by i^3 rotates a complex number by $270°$ anti-clockwise
Multiplication by i^4 rotates a complex number by $360°$ anti-clockwise

Multiplication by $-i, -i^2, -i^3 and -i^4$ reverses the direction of the rotation to clockwise

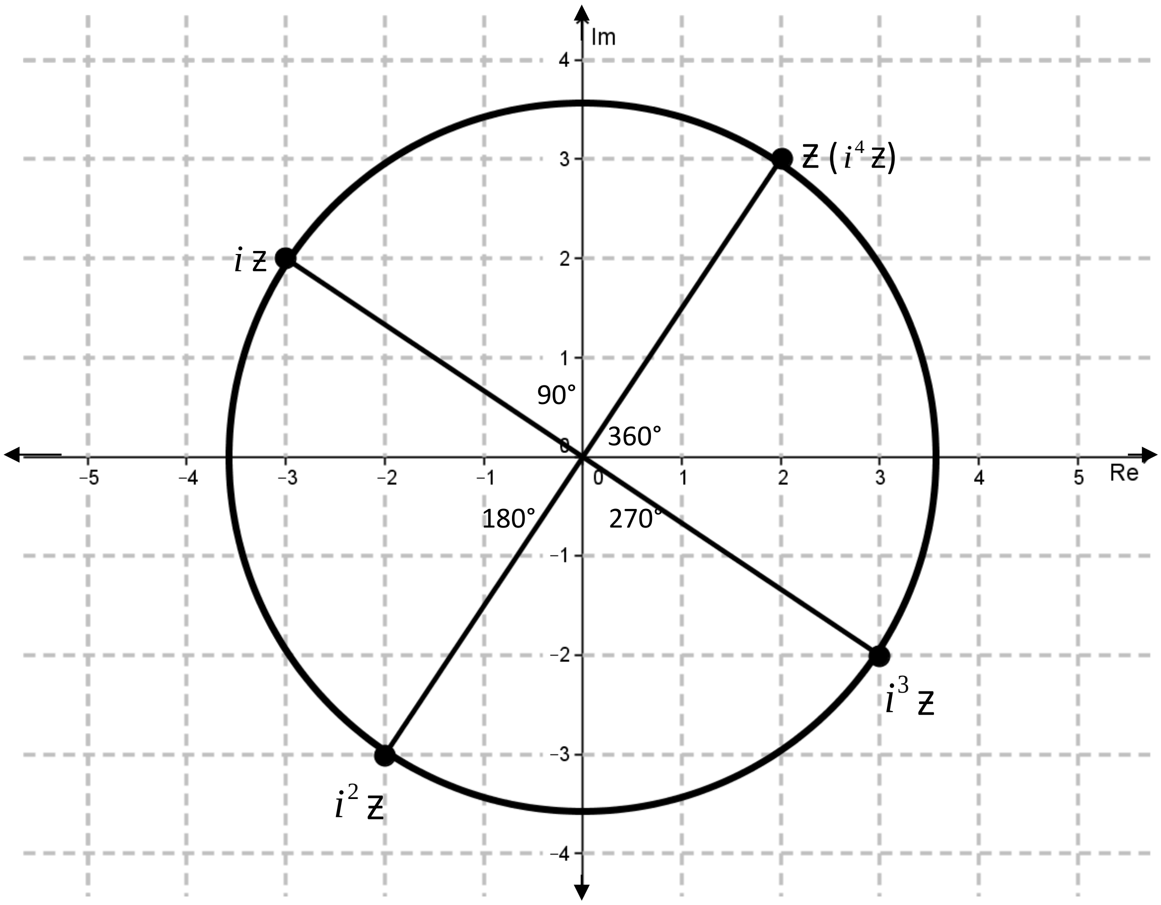
N.B. $i^2 = -1,\ i^3 = -i \text{ and } i^4 = 1$

Q. $z = 2 + 3i.$

i) Represent $z,\ iz,\ i^2z,\ i^3z \text{ and } i^4z$ on an Argand diagram.

ii) Using the origin as the centre point, draw a circle through the complex numbers $z,\ iz,\ i^2z,\ i^3z \text{ and } i^4z$. What do you notice?

iii) Verify that $|z| = |iz| = |i^2z| = |i^3z| = |i^4z|$, i.e., prove that all the points are the same distance from the origin. $(Modulus = Radius)$


Im
Re
z (i⁴z)
i z
i² z
i³ z
90°
360°
180°
270°
4
3
2
1
0
−1
−2
−3
−4
−5
−4
−3
−2
−1
0
1
2
3
4
5

$z = 2 + 3i$

$|z| = |2+3i| = \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$

$iz = i(2+3i) = 2i + 3i^2 = 2i + 3(-1) = -3 + 2i$

$|iz| = |-3+2i| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$

$i^2 z = -1(2+3i) = -2 - 3i$

$|i^2 z| = |-2-3i| = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

$i^3 z = -i(2+3i) = -2i - 3i^2 = -2i - 3(-1) = 3 - 2i$

$|i^3 z| = |3-2i| = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

$i^4 z = 1(2+3i) = 2 + 3i = z$

$|i^4 z| = |z| = |2+3i| = \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$

$\therefore \textit{Modulus} = \textit{Radius Length} = \sqrt{13}$

2. Translations

A **translation** is the image of an object by moving every point of the object in the same direction and same distance away, without rotating or resizing the object; simply changing the location of the object.

If you **add a given complex number** to each complex number that makes up an object you will **translate** that object and **move** it to a **different location** to create an image.

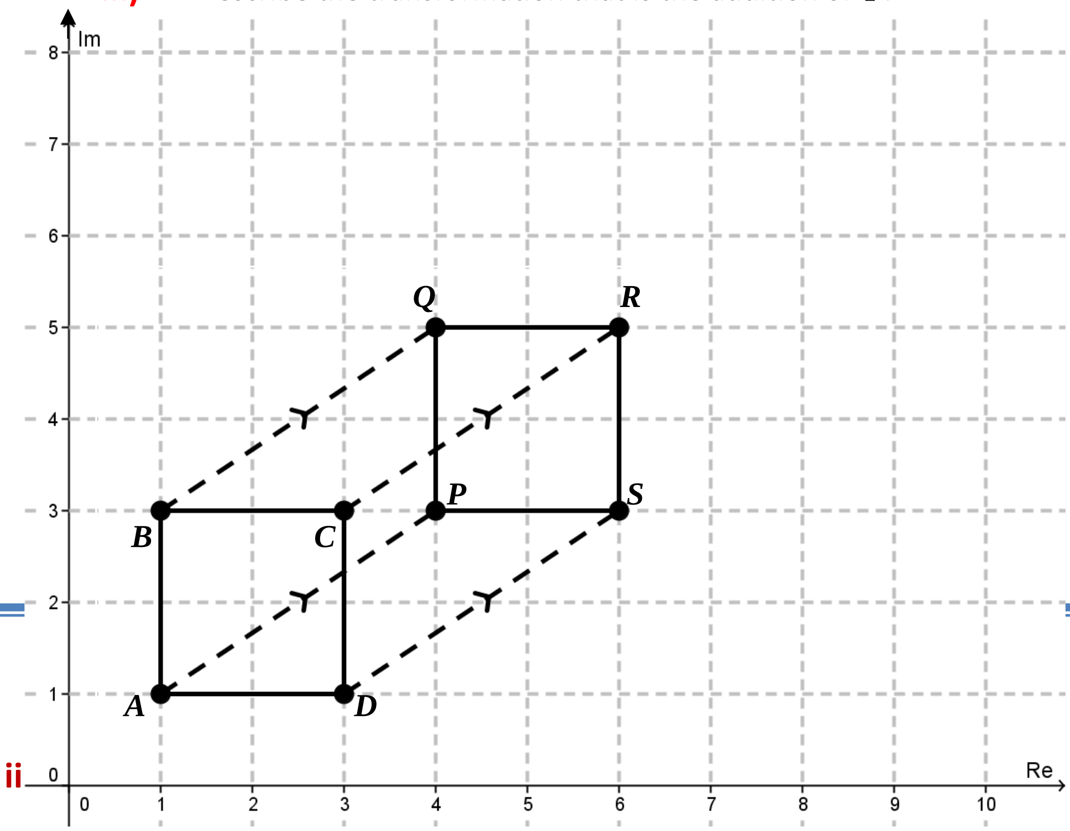
$\therefore$ *the addition of complex numbers means that you are translating / moving them on an Argand diagram.*

Q The four complex numbers $A(1+i), B(1+3i), C(3+3i)$ *and* $D(3+i)$ form the vertices of a square.

i) Plot the complex numbers on an Argand diagram (complex plane).

ii) If $z = 3 + 2i$, evaluate and plot the points on an Argand diagram: $P = A + z, Q = B + z, R = C + z$ *and* $S = D + z$.

iii) Describe the transformation that is the addition of z.



$P = A + z$

$P = (1+i)+(3+2i)$
$P = 3+3i$

$Q = B + z$

$Q = (1+3i)+(3+2i)$
$Q = 4+5i$

$R = C + z$

$R = (3+3i)+(3+2i)$
$R = 6+5i$

$S = D + z$

$S = (3+i)+(3+2i)$
$S = 6+3i$

iii) The transformation that maps the square $ABCD$ onto the quadrilateral $PQRS$ is a ***translation***, where all the points **A, B, C** and **D** are all **moved the same distance and in the same direction** on the complex plane.

A **dilation** is the resizing of an object, making it larger or smaller.

If a **complex number** is **multiplied** by a **real number (scalar)**, then its **modulus** (distance from the origin) will be **multiplied by this scalar.**

If $-1 > real\ number > 1$, then the dilation is referred to as a ***stretching*** and the object is ***enlarged.***

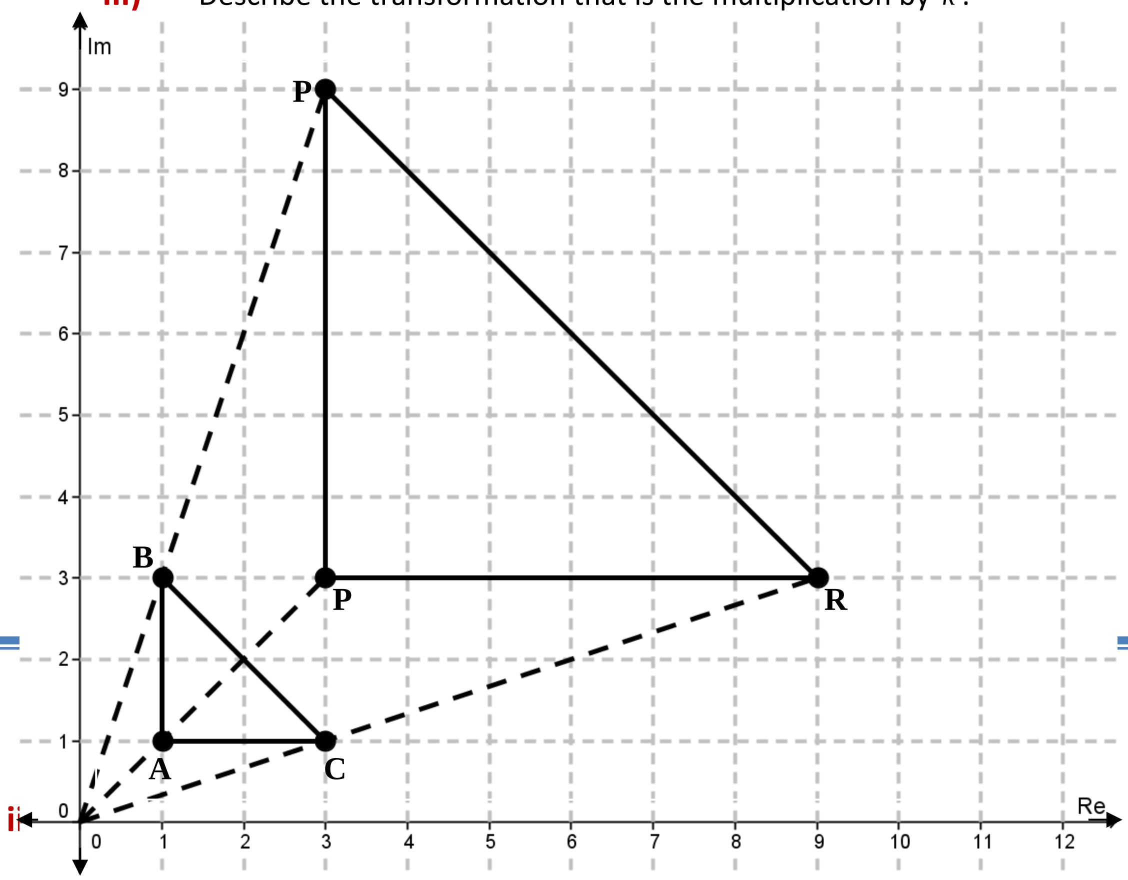
If $-1 < real\ number < 1$, then the dilation is referred to as a ***contracting*** and the object is ***reduced***.

Q. The three points $A(1+i), B(1+3i)\ and\ C(3+i)$ are vertices of a triangle.

i) Plot the complex numbers on an Argand diagram (complex plane).

ii) If $k=3$, evaluate and plot the points on an Argand diagram: $P=kA,\ Q=kB\ and\ R=kC$.

iii) Describe the transformation that is the multiplication by k.



$P = kA$

$P = 3(1+i)$
$P = 3+3i$

$Q = kB$

$Q = 3(1+3i)$
$Q = 3+9i$

$R = kC$

$R = 3(3+i)$
$R = 9+3i$

iii) From the diagram we see that all the points A, B and C are moved further from the origin by a factor of $k=3$. We call the transformation that maps the triangle ABC onto the triangle PQR a ***dilation*** by a factor of 3. The points A, B and C are said to be stretching the complex plane and the triangle PQR is an enlargement of the triangle ABC.

N.B. If you multiply a complex number by $2i$, its modulus will be doubled and it will be rotated by $90°$.

N.B. If z is a complex number, then $|kz| = k|z|$, where k is a real number.

Example ⑤. Transformations

Q1 $z_1 = 2+4i,\ z_2 = 2+3i,\ z_3 = -1+2i$ and $w = 1+i$.

(i) Plot the points $z_1,\ z_2$ and z_3 on an Argand diagram.

(ii) Evaluate $z_1 + w,\ z_2 + w$ and $z_3 + w$ and plot the answers on the Argand diagram

(iii) Describe the transformation that is the addition of w.

Q2 $z_1 = 2+4i,\ z_2 = 2+3i,\ z_3 = -1+2i$ and $k = 2$.

(i) Plot the points $z_1,\ z_2$ and z_3 on an Argand diagram.