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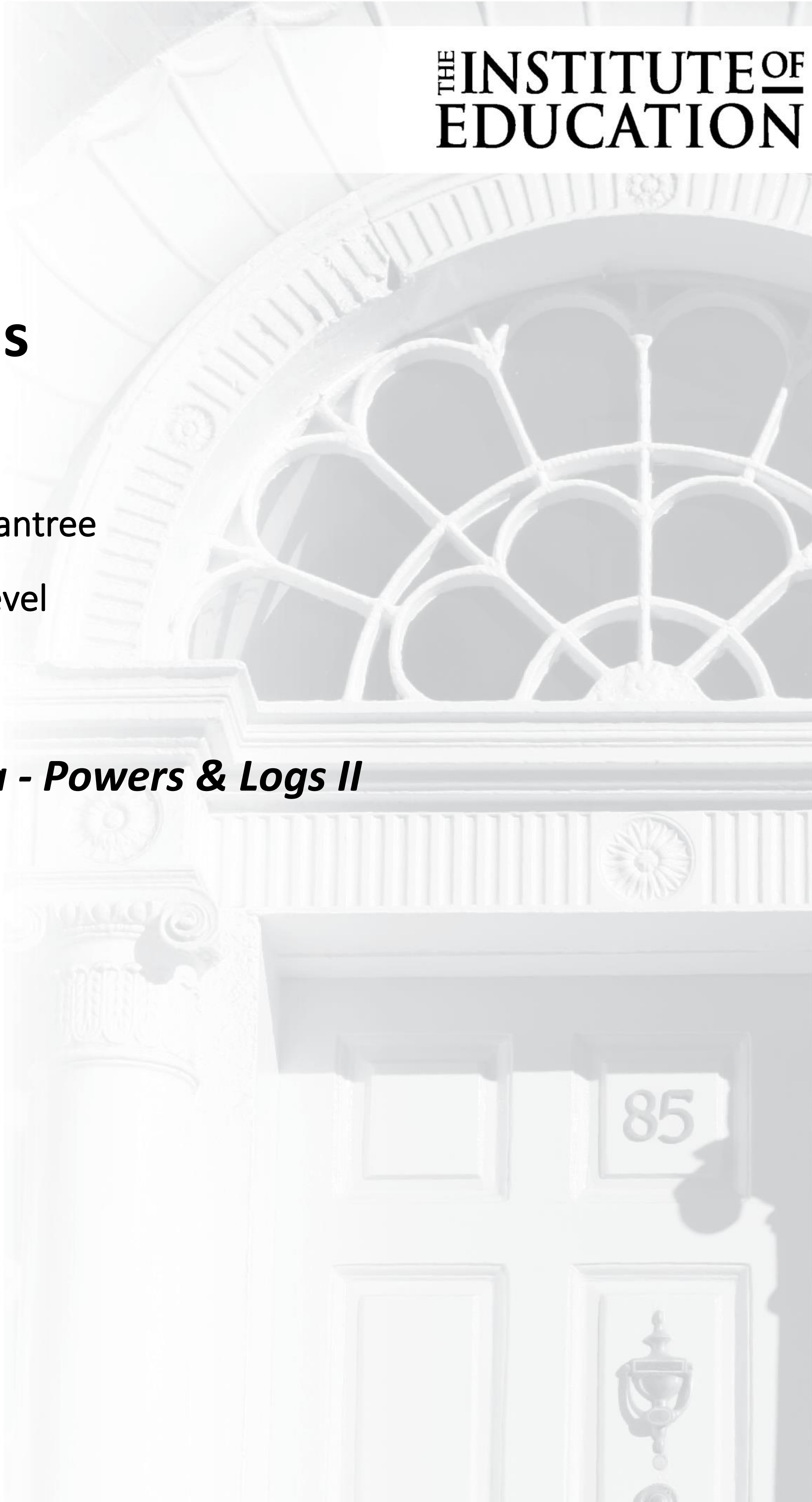

Maths

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Higher Level

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Algebra - Powers & Logs II

Unit 8

HIGHER MATHS SAMPLE

Algebra: Powers and Logs

8.2 Logarithms

1. Definition of Logarithm (Log)

* Logarithm, or log, is another term for index, when the emphasis is placed on the index.
* For example, in the index statement

$$2^3 = 8,$$

the log is 3.

2. Index Statement and Log Statement

* The index statement

$$2^3 = 8$$

can be written as a log statement in the form

$$\log_2 8 = 3,$$

i.e. the base, 2, is written as a subscript and the value of the power, 8, is written directly after the log, and the value of the log (index) is 3.

* The meaning of the symbol

$$\log_a x$$

is the index that must be placed above the base *a* to give the value *x*.

Definition of Logs

$$a^y = x \quad \Leftrightarrow \quad \log_a x = y$$

* It is important to be able to convert between an index statement ($a^y = x$) and the corresponding log statement ($\log_a x = y$).

* For example:

$$(2x^2 + 5x)^8 = 3y + 2 \quad \text{... index statement}$$

can be written in the form

$$\log_{(2x^2+5x)}(3y+2) = 8 \quad \text{... log statement.}$$

3. Laws of Logs

Laws of Logs

1. $\log_a(xy) = \log_a x + \log_a y$

2. $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

3. $\log_a x^q = q\log_a x$

4. $\log_a a = 1$ and $\log_a 1 = 0$

5. $\log_a x = \dfrac{\log_b x}{\log_b a}$ **(Change of Base Law)**

The Laws of Logs are related to the Laws of Indices and can be derived from these laws. These laws are often used in conjunction with each other.

Example 1

If $x = \log_{10} 2$ and $y = \log_{10} 3$, express in terms of *x* and *y*:

(i) $\log_{10}\sqrt{\dfrac{32}{27}}$, (ii) $\log_{10}\dfrac{4000}{9}$.

Solution

(i) $\log_{10}\sqrt{\dfrac{32}{27}} = \log_{10}\left(\dfrac{32}{27}\right)^{\frac{1}{2}}$

$= \dfrac{1}{2}\log_{10}\dfrac{32}{27}$ … *Law 3*

$= \dfrac{1}{2}(\log_{10} 32 - \log_{10} 27)$ … *Law 2*

$= \dfrac{1}{2}\left(\log_{10} 2^5 - \log_{10} 3^3\right)$

$= \dfrac{1}{2}(5\log_{10} 2 - 3\log_{10} 3)$ … *Law 3*

$= \dfrac{1}{2}(5x - 3y)$

(ii) $\log_{10}\dfrac{4000}{9} = \log_{10} 4000 - \log_{10} 9$ … *Law 2*

$= \log_{10}(1000\times 4) - \log_{10} 3^2$

$= \log_{10} 10^3 + \log_{10} 2^2 - \log_{10} 3^2$ … *Law 1*

$= 3\log_{10} 10 + 2\log_{10} 2 - 2\log_{10} 3$ … *Law 3*

$= 3 + 2x - 2y.$ … *Law 4*

Example 2

(i) Show that $\log_a b = \dfrac{1}{\log_b a}$.

(ii) Show that $\dfrac{1}{\log_a x} + \dfrac{1}{\log_b x} + \dfrac{1}{\log_c x} = \log_x (abc)$.

Solution

(i) By Law 5, the Change of Base Law,

$$\log_a b = \frac{\log_b b}{\log_b a}$$

$$= \frac{1}{\log_b a} \qquad \textit{... Law 4}$$

(ii) By part (i),

$$\frac{1}{\log_a x} = \log_x a\,, \quad \frac{1}{\log_b x} = \log_x b, \quad \frac{1}{\log_c x} = \log_x c$$

Then

$$\frac{1}{\log_a x} + \frac{1}{\log_b x} + \frac{1}{\log_c x} = \log_x a + \log_x b + \log_x c$$

$$= \log_x (a \times b \times c)$$

$$= \log_x (abc).$$

4. Natural Logs

* Logs with a base of e, which is approximately $2\cdot718$, are called natural logs, as they occur frequently in nature.
* An example of a natural log is
  $\log_e x$.
* $\log_e$ can also be written as 'ln'. Thus we can write
  $\log_e x = \ln x$.
* Note that $\ln e = \log_e e = 1$.
* Natural logs obey all the usual Laws of Logs, e.g.
  (i) $\ln(x.y) = \ln x + \ln y$ (ii) $\ln x^4 = 4 \ln x$.

Example 3

Show that $\ln\left(xe^{2x}\right) = 2x + \ln x$.

Solution

$$\ln\left(xe^{2x}\right) = \ln x + \ln e^{2x}$$

$$= \ln x + 2x \ln e$$

$$= 2x + \ln x.$$

5. Using a Calculator to Evaluate Logs

* On scientific calculators, there are three separate log function buttons.

* The [log] function is used to calculate logs with a base of 10, e.g. to find $\log_{10} 7\cdot 52$, press [log] $7\cdot 52$ [)] [=] to get $0\cdot 8762$.

* The [ln] function is used to calculate natural logs, i.e. $\log_e$, for example, to find $\ln 3\cdot 845$, press [ln] $3\cdot 845$ [)] [=] to get $1\cdot 34677$.

* The [$\log_{\square}\square$] function is used to calculate any log, e.g. to find $\log_{1\cdot 05} 1\cdot 89$, press [$\log_{\square}\square$] $1\cdot 05$ [$\rightarrow$] $1\cdot 89$ [=] to get $13\cdot 047$.

Exercises 8.2

1. Write each of the following as a log statement:

(i) $3^5 = 243$ (ii) $7^4 = 2401$

(iii) $1\cdot 2^6 = 2\cdot 985984$ (iv) $3\cdot 5^{4\cdot 2} = 192\cdot 7905695$

2. Write each of the following as an index statement:

(i) $\log_5 125 = 3$ (ii) $\log_3 6561 = 8$

(iii) $\log_{1\cdot 05} 1\cdot 795856326 = 12$ (iv) $\log_{2\cdot 1} 438\cdot 7308937 = 8\cdot 2$

Write each of the following as a single log.

3. $\log_2 x + \log_2 y$

4. $\log_3 a^2 - \log_3 b$

5. $\log_3 a - 2\log_3 b + 3\log_3 c$

6. $\log_4 x + \frac{1}{2}\log_4 y - 3\log_4 z$

7. If $x = \log_2 a$, express in terms of x:

(i) $\log_2 a^3$ (ii) $\log_2 16a^2$ (iii) $\log_2 \frac{a^4}{32}$.

8. If $p = \log_{27} q$, express in terms of p:

(i) q (ii) $\log_9 q$ (iii) $\log_q 9$.

9. If $\log_a 2 = s$ and $\log_a 3 = t$, where $a > 0$, express each of the following in terms of s and t:

(i) $\log_a 108$ (ii) $\log_a 48a^3$ (iii) $\log_a \frac{27}{32a^4}$.

10. (i) If $\log_a x = \log_a y$, show that $x = y$.

(ii) Hence show that if

$$\log_4 x = \log_2 y$$

then $x = y^2$.

11. Write each of the following as a log statement:
(i) $e^{2t} = p$ (ii) $e^{1+3x} = 4y$ (iii) $e^{x^2-x} = 3\cdot 2$.

12. Write each of the following as an index statement:
(i) $\ln(x^2 - 3x) = y$ (ii) $\ln(3x+7) = 2\cdot 8$
(iii) $\ln(5t-1) = 0\cdot 15s$ (iv) $\ln(2a+b) = 2\cdot 19$.

Simplify each of the following expressions.

13. $\log_e\left(x^2 e^{-x}\right)$ **14.** $\log_e e^{2+\sin x}$ **15.** $\log_e\left(x^4 e^{\cos x}\right)$

16. $\ln(x^2\sqrt{x+1})$ **17.** $\ln(x^3 e^{\sin x})$ **18.** $\ln\left(\frac{\sqrt{x}}{e^{4x}}\right)$.

8.3 Equations Involving Laws of Indices and Logs

A Type 1: Constant Index

1. Method
To solve for a an equation such as

$$a^p = b,$$

especially where p is not just 2 or 3, we can proceed as follows:

* Put both sides of the equation to the power of $\frac{1}{p}$:

$$a^p = b$$
$$\left(a^p\right)^{\frac{1}{p}} = (b)^{\frac{1}{p}}$$

* As the index on the LHS is now 1, we get:

$$a = b^{\frac{1}{p}}.$$

Example 1
Solve the equation

$$(3+2x)^{7\cdot 3} = 141\cdot 812,$$

giving your answer correct to three decimal places.
Solution

$$(3+2x)^{7\cdot 3} = 141\cdot 812$$
$$3+2x = 141\cdot 812^{\frac{1}{7\cdot 3}}$$
$$3+2x = 1\cdot 9713 \quad \text{... by calculator}$$
$$2x = -1\cdot 0287$$
$$x = -0\cdot 514$$

B Type 2: Unknown in Index: Common Base Possible

1. Common Base Possible: Single Power = Single Power

* Some equations with the unknown in the index can be written with a single power on each side of the equation.

* With some of these, it is possible to write each side with the same base.

* For example, the equation

$$2^x = \frac{1}{64}$$

can be written

$$2^x = 2^{-6}.$$

* In this case, we can put the indices equal to get

$$x = -6.$$

* Alternatively, the equation $2^x = \frac{1}{64}$ can be converted to log form:

$$x = \log_2 \frac{1}{64} = -6.$$

Example 2

Solve for $x \in \mathbb{R}$, $\quad 8^{1-2x} = \sqrt{\dfrac{1}{32^x}}.$

Solution

$$(2^3)^{1-2x} = (32^{-x})^{\frac{1}{2}}$$

$$2^{3-6x} = \left((2^5)^{-x}\right)^{\frac{1}{2}}$$ *... write each side with a base of 2*

$$2^{3-6x} = (2^{-5x})^{\frac{1}{2}}$$

$$2^{3-6x} = 2^{-\frac{5x}{2}}$$ *... now each side is a power with the same base*

$$3 - 6x = -\frac{5x}{2}$$ *... equating indices*

$$6 - 12x = -5x$$

$$6 = 7x$$

$$x = \frac{6}{7}$$

2. Evaluating Logs Without Using a Calculator

If the base of a log and the value of the power can be written in terms of the same base, the log can be calculated without using a calculator.

Example 3
Evaluate, without using a calculator, $\log_3 \sqrt{27}$.
Solution
Let $\log_3 \sqrt{27} = x$.
Then $3^x = \sqrt{27}$

$3^x = \left(3^3\right)^{\frac{1}{2}}$

$3^x = 3^{\frac{3}{2}}$

$x = \frac{3}{2}$.

C Type 3: Unknown in Index: Common Base Not Possible

1. Using Logs to Solve Index Equations

* Logs can be used to solve equations with the unknown in the index, if the base and the value cannot be easily expressed as powers with the same base.

* For example, the equation
$$7^{4x-3} = 5$$
can be solved by using logs, because it is not possible to find a common base for the 7 and the 5.

* Two different approaches exist: converting to a log statement and taking logs on both sides. They are shown in the example below.

Example 4
Solve the equation
$$7^{4x-3} = 5$$
for $x \in \mathbb{R}$. Give your answer correct to three decimal places.
Solution
Method 1: Convert to a Log Statement

$7^{4x-3} = 5$

$\log_7 5 = 4x - 3$ ... *converting to a log statement*

$0 \cdot 8271 = 4x - 3$ ... *using the* $\log_{\square}\square$ *function on your calculator*

$4x = 3 \cdot 8271$

$x = 0 \cdot 957$

Method 2: Taking Logs (natural logs) on Both Sides

$7^{4x-3} = 5$

$\ln 7^{4x-3} = \ln 5$ ... *taking* ln *on each side*
$(4x-3)\ln 7 = \ln 5$
$4x-3 = \dfrac{\ln 5}{\ln 7}$
$4x-3 = 0\cdot 8271$
$4x = 3\cdot 8271$
$x = 0\cdot 957$.

2. More Advanced Equations

* If an equation contains unknowns in two separate powers, with no common base possible, it is more convenient to take logs on both sides to find the solution.

* Example 5 shows one of these equations.

Example 5
Solve the equation

$$4500(1\cdot 045)^t = 6300(1\cdot 035)^{t+2}$$

for $t \in \mathbb{R}$. Give your answer correct to three decimal places.
Solution

$4500(1\cdot 045)^t = 6300(1\cdot 035)^{t+2}$
$(1\cdot 045)^t = 1\cdot 4(1\cdot 035)^{t+2}$
$\ln(1\cdot 045)^t = \ln 1\cdot 4(1\cdot 035)^{t+2}$
$t\ln 1\cdot 045 = \ln 1\cdot 4 + \ln(1\cdot 035)^{t+2}$
$t\ln 1\cdot 045 = \ln 1\cdot 4 + (t+2)\ln 1\cdot 035$
$0\cdot 04402t = 0\cdot 33647 + (t+2)(0\cdot 03440)$
$0\cdot 04402t = 0\cdot 33647 + 0\cdot 03440t + 0\cdot 06880$
$0\cdot 00962t = 0\cdot 40527$
$t = 42\cdot 128$

D Type 4: Equations which Become Quadratic

1. Equations with Three or More Terms

* An equation such as

$$2^{2x+3} - 33(2^x) + 4 = 0$$

can be converted to a quadratic equation by letting y be the basic power present, i.e.

$$y = 2^x .$$

* To write 2^{2x+3} in terms of $y = 2^x$:

$$2^{2x+3} = 2^{2x}.2^3$$
$$= (2^x)^2(8)$$
$$= 8y^2$$

* We can now write the given equation as

$$8y^2 - 33y + 4 = 0.$$

* We now solve this equation for *y*. Don't forget to finish by calculating the corresponding values of *x*.

Example 6

Solve the equation

$$e^{2x+1} - 14\cdot1e^x + 2\cdot7 = 0$$

for $x \in \mathbb{R}$. Give your answers correct to two decimal places.

Solution

Let $y = e^x$.

Then $e^{2x+1} = e^{2x}.e^1$

$$= e(e^x)^2$$
$$= e\,y^2.$$

Then the equation can be written:

$$e\,y^2 - 14\cdot1\,y + 2\cdot7 = 0 \qquad [a = e,\ b = -14\cdot1,\ c = 2\cdot7]$$

$$y = \frac{14\cdot1 \pm \sqrt{(-14\cdot1)^2 - 4(e)(2\cdot7)}}{2(e)}$$

$$y = \frac{14\cdot1 \pm 13\cdot017}{2e}$$

$y = 4\cdot988$ or $y = 0\cdot199$

$e^x = 4\cdot988$ or $e^x = 0\cdot199$

$x = \ln 4\cdot988$ or $x = \ln 0\cdot199$

$x = 1\cdot61$ or $x = -1\cdot61$.

Exercises 8.3

Solve the following equations for $x \in \mathbb{R}$. Give your answers correct to two decimal places.

1. $x^6 = 80$ **2.** $x^{5\cdot2} = 11\cdot7$ **3.** $(x+3)^5 = 21\cdot8$

4. $(x+5)^7 = 98$ **5.** $(2x-1)^{1\cdot3} = 273$ **6.** $(3x+2)^{4\cdot6} = 823\cdot6$

Solve the following equations for $x \in \mathbb{R}$.

7. $2^x = 32$ **8.** $3^x = \frac{1}{27}$ **9.** $2^{3x-1} = \frac{1}{64}$

10. $3^{2x+1} = \sqrt{27}$ **11.** $9^{x+2} = \frac{1}{27^{2x+5}}$ **12.** $4^{2x-1} = \sqrt{\frac{2}{8^{x+3}}}$

Without using a calculator, evaluate each of the following logs.

13. $\log_4 16$ **14.** $\log_3 \sqrt{27}$ **15.** $\log_4 128$ **16.** $\log_{16} \sqrt{128}$

Solve the following equations for $x \in \mathbb{R}$. Where appropriate, give your answers correct to two decimal places.

17. $2 \cdot 6^x = 9 \cdot 47$ **18.** $5 \cdot 18^x = 167$ **19.** $0 \cdot 92^x = 0 \cdot 05$

20. $3 \cdot 82^{2x+1} = 39 \cdot 7$ **21.** $1 \cdot 47^{3x-1} = 12 \cdot 8$ **22.** $10 \cdot 4^{2-x} = 43 \cdot 7$

23. $e^{2x+1} = 65 \cdot 9$ **24.** $e^{3x+4} = 87 \cdot 23$ **25.** $8 \cdot 9e^{3x+7} = 241 \cdot 87$

26. Solve the equation
$$7(2^{x+5}) = 1 \cdot 4(2^{2x+1})$$
for $x \in \mathbb{R}$. Give your answer correct to two decimal places.

27. Solve the equation
$$1 \cdot 3 \times 10^4 \times e^{3t-1} = 6 \cdot 5 \times 10^5 \times e^{t+3}$$
for $t \in \mathbb{R}$. Give your answer correct to two decimal places.

28. Solve the equation
$$8 \cdot 9(e^{4x+7}) = 32 \cdot 5(e^{3x-1})$$
for $x \in \mathbb{R}$. Give your answer correct to two decimal places.

29. Solve the equation
$$3 \cdot 2(2 \cdot 56)^x = 18 \cdot 9(1 \cdot 73)^{x+2}$$
for $x \in \mathbb{R}$. Give your answer correct to two decimal places.

30. Solve the equation
$$2^{2x} - 20(2^x) + 64 = 0,$$
for $x \in \mathbb{R}$.

31. Solve the equation
$$3^{2x+5} - 4(3^{x+2}) + 1 = 0$$
for $x \in \mathbb{R}$.

32. Solve the equation
$$3^{2x+1} - 28(3^x) + 9 = 0,$$
for $x \in \mathbb{R}$.

33. Solve the equation
$$\frac{2}{e^x} = e^x - 1$$
for $x \in \mathbb{R}$, correct to two decimal places.

Solutions to Exercises

Exercises 8.2

1. (i) $3^5 = 243$

$\log_3 243 = 5$

(ii) $7^4 = 2401$

$\log_7 2401 = 4$

(iii) $1\cdot 2^6 = 2\cdot 985984$

$\log_{1\cdot 2} 2\cdot 985984 = 6$

(iv) $3\cdot 5^{4\cdot 2} = 192\cdot 7905695$

$\log_{3\cdot 5} 192\cdot 7905695 = 4\cdot 2$

2. (i) $\log_5 125 = 3$

$5^3 = 125$

(ii) $\log_3 6561 = 8$

$3^8 = 6561$

(iii) $\log_{1\cdot 05} 1\cdot 795856326 = 12$

$1\cdot 05^{12} = 1\cdot 795856326$

(iv) $\log_{2\cdot 1} 438\cdot 7308937 = 8\cdot 2$

$2\cdot 1^{8\cdot 2} = 438\cdot 7308937$

3. $\log_2 x + \log_2 y = \log_2 xy$

4. $\log_3 a^2 - \log_3 b = \log_3 \dfrac{a^2}{b}$

5. $\log_3 a - 2\log_3 b + 3\log_3 c$

$= \log_3 a - \log_3 b^2 + \log_3 c^3$

$= \log_3 \dfrac{ac^3}{b^2}$

6. $\log_4 x + \dfrac{1}{2}\log_4 y - 3\log_4 z$

$= \log_4 x + \log_4 y^{\frac{1}{2}} - \log_4 z^3$

$= \log_4 \dfrac{x\sqrt{y}}{z^3}$

7. (i) $\log_2 a^3 = 3\log_2 a = 3x$

(ii) $\log_2 16a^2 = \log_2 16 + \log_2 a^2$

$= \log_2 2^4 + 2\log_2 a$

$= 4\log_2 2 + 2x = 4 + 2x$

(iii) $\log_2 \dfrac{a^4}{32} = \log_2 a^4 - \log_2 32$

$= 4\log_2 a - \log_2 2^5$

$= 4x - 5\log_2 2 = 4x - 5$

8. (i) $p = \log_{27} q$

$q = 27^p$

(ii) $\log_9 q = \log_9 27^p = p\log_9 27$

Let $\log_9 27 = x$

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = \dfrac{3}{2}$

Then

$\log_9 q = \dfrac{3}{2}p$

(iii)

$\log_q 9 = \dfrac{\log_9 9}{\log_9 q} = \dfrac{1}{\log_9 q} = \dfrac{1}{\frac{3}{2}p} = \dfrac{2}{3p}$.

9. (i) $\log_a 108 = \log_a(4\times 27)$

$= \log_a 2^2 + \log_a 3^3$

$= 2\log_a 2 + 3\log_a 3$

$= 2s + 3t$

(ii) $\log_a 48a^3 = \log_a 48 + \log_a a^3$

$= \log_a(16\times 3) + 3\log_a a$

$= \log_a 2^4 + \log_a 3 + 3$

$= 4\log_a 2 + t + 3$

$= 4s + t + 3$

(iii) $\log_a \dfrac{27}{32a^4} = \log_a 27 - \log_a 32a^4$

$= \log_a 3^3 - \log_a 2^5 - \log_a a^4$

$= 3\log_a 3 - 5\log_a 2 - 4\log_a a$

$= 3t - 5s - 4$

10. (i) $\log_a x = \log_a y$

$\log_a x - \log_a y = 0$

$\log_a \frac{x}{y} = 0$

$\frac{x}{y} = a^0 = 1$

$x = y$

(ii) $\log_4 x = \log_2 y$

$\log_4 x = \frac{\log_4 y}{\log_4 2}$

$\log_4 x = \frac{\log_4 y}{\frac{1}{2}} = 2\log_4 y$

$\log_4 x = \log_4 y^2$

$x = y^2$

11. (i) $e^{2t} = p$

$\log_e p = 2t$

$\ln p = 2t$

(ii) $e^{1+3x} = 4y$

$\log_e 4y = 1 + 3x$

$\ln 4y = 1 + 3x$

(iii) $e^{x^2 - x} = 3\cdot 2$

$\log_e 3\cdot 2 = x^2 - x$

$\ln 3\cdot 2 = x^2 - x$

12. (i) $\ln(x^2 - 3x) = y$

$e^y = x^2 - 3x$

(ii) $\ln(3x+7) = 2\cdot 8$

$e^{2\cdot 8} = 3x + 7$

(iii) $\ln(5t-1) = 0\cdot 15x$

$e^{0\cdot 15x} = 5t - 1$

(iv) $\ln(2a+b) = 2\cdot 19$

$e^{2\cdot 19} = 2a + b$

13. $\log_e (x^2 e^{-x}) = \log_e x^2 + \log_e e^{-x}$

$= 2\log_e x + (-x)\log_e e$

$= 2\log_e x - x$

14. $\log_e e^{\sin x + 2} = (\sin x + 2)\log_e e$

$= \sin x + 2$

15. $\log_e (x^4 e^{\cos x}) = \log_e x^4 + \log_e e^{\cos x}$

$= 4\log_e x + (\cos x)\log_e e$

$= 4\log_e x + \cos x$

16. $\ln(x^2\sqrt{x+1}) = \ln x^2 + \ln(x+1)^{\frac{1}{2}}$

$= 2\ln x + \frac{1}{2}\ln(x+1)$

17. $\ln(x^3 e^{\sin x}) = \ln x^3 + \ln e^{\sin x}$

$= 3\ln x + \sin x(\ln e)$

$= 3\ln x + \sin x$

18. $\ln\left(\frac{\sqrt{x}}{e^{4x}}\right) = \ln\sqrt{x} - \ln e^{4x}$

$= \ln x^{\frac{1}{2}} - (4x)\ln e$

$= \frac{1}{2}\ln x - 4x$

Exercises 8.3

1. $x^6 = 80$

$x = 80^{\frac{1}{6}} = 2\cdot 08$

2. $x^{5\cdot 2} = 11\cdot 7$

$x = 11\cdot 7^{\frac{1}{5\cdot 2}} = 1\cdot 60$

3. $(x+3)^5 = 21\cdot 8$

$x + 3 = 21\cdot 8^{\frac{1}{5}} = 1\cdot 85$

$x = -1\cdot 15$

4. $(x+5)^7 = 98$

$x + 5 = 98^{\frac{1}{7}} = 1\cdot 93$

$x = -3\cdot 07$

5. $(2x-1)^{1\cdot 3} = 273$

$2x - 1 = 273^{\frac{1}{1\cdot 3}} = 74\cdot 812$

$2x = 75\cdot 812$

$x = 37\cdot 91$

6. $(3x+2)^{4\cdot 6} = 823\cdot 6$

$3x + 2 = 823\cdot 6^{\frac{1}{4\cdot 6}} = 4\cdot 303$

$3x = 2\cdot 303$

$x = 0\cdot 77$

7. $2^x = 32$

$2^x = 2^5$

$x = 5$

8. $3^x = \frac{1}{27}$

$3^x = \frac{1}{3^3}$

$3^x = 3^{-3}$

$x = -3$

9. $2^{3x-1} = \frac{1}{64}$

$2^{3x-1} = \frac{1}{2^6}$

$2^{3x-1} = 2^{-6}$

$3x - 1 = -6$

$3x = -5$

$x = -\frac{5}{3}$

10. $3^{2x+1} = \sqrt{27}$

$3^{2x+1} = (3^3)^{\frac{1}{2}}$

$3^{2x+1} = 3^{\frac{3}{2}}$

$2x + 1 = \frac{3}{2}$

$2x = \frac{1}{2}$

$x = \frac{1}{4}$

11. $9^{x+2} = \frac{1}{27^{2x+5}}$

$(3^2)^{x+2} = \frac{1}{(3^3)^{2x+5}}$

$3^{2x+4} = \frac{1}{3^{6x+15}}$

$3^{2x+4} = 3^{-6x-15}$

$2x + 4 = -6x - 15$

$8x = -19$

$x = -\frac{19}{8}$

12. $4^{2x-1} = \sqrt{\frac{2}{(2^3)^{x+3}}}$

$(2^2)^{2x-1} = \sqrt{\frac{2}{2^{3x+9}}}$

$2^{4x-2} = \sqrt{2^{1-(3x+9)}}$

$2^{4x-2} = (2^{-3x-8})^{\frac{1}{2}}$

$2^{4x-2} = 2^{\frac{1}{2}(-3x-8)}$

$4x - 2 = \frac{1}{2}(-3x - 8)$

$8x - 4 = -3x - 8$

$11x = -4$

$x = -\frac{4}{11}$

13. $\log_4 16 = x$

$4^x = 16$

$4^x = 4^2$

$x = 2$

14. $\log_3 \sqrt{27} = x$

$3^x = (3^3)^{\frac{1}{2}}$

$3^x = 3^{\frac{3}{2}}$

$x = \frac{3}{2}$

15. $\log_4 128 = x$

$(2^2)^x = 2^7$

$2^{2x} = 2^7$

$2x = 7$

$x = \frac{7}{2}$

16. $\log_{16} \sqrt{128} = x$

$(2^4)^x = (2^7)^{\frac{1}{2}}$

$2^{4x} = 2^{\frac{7}{2}}$

$4x = \frac{7}{2}$

$x = \frac{7}{8}$

17. $2\cdot 6^x = 9\cdot 47$

$x = \log_{2\cdot 6} 9\cdot 47$

$x = 2\cdot 35$

18. $5\cdot 18^x = 167$

$x = \log_{5\cdot 18} 167$

$x = 3\cdot 11$

19. $0\cdot 92^x = 0\cdot 05$

$\log_{0\cdot 92} 0\cdot 05 = x$

$x = 35\cdot 92$

20. $3\cdot 82^{2x+1} = 39\cdot 7$

$\log_{3\cdot 82} 39\cdot 7 = 2x + 1$

$2\cdot 747 = 2x + 1$

$2x = 1\cdot 747$

$x = 0\cdot 87$

21. $1\cdot 47^{3x-1}=12\cdot 8$
$\log_{1\cdot 47}12\cdot 8=3x-1$
$6\cdot 617=3x-1$
$3x=7\cdot 617$
$x=2\cdot 54$

22. $10\cdot 4^{2-x}=43\cdot 7$
$\log_{10\cdot 4}43\cdot 7=2-x$
$1\cdot 613=2-x$
$x=0\cdot 39$

23. $e^{2x+1}=65\cdot 9$
$\ln 65\cdot 9=2x+1$
$4\cdot 188=2x+1$
$2x=3\cdot 188$
$x=1\cdot 59$

24. $e^{3x+4}=87\cdot 23$
$\ln 87\cdot 23=3x+4$
$4\cdot 469=3x+4$
$3x=0\cdot 469$
$x=0\cdot 16$

25. $8\cdot 9e^{3x+7}=241\cdot 87$
$e^{3x+7}=27\cdot 1764$
$\ln 27\cdot 1764=3x+7$
$3\cdot 302=3x+7$
$3x=-3\cdot 697$
$x=-1\cdot 23$

26. $7(2^{x+5})=1\cdot 4(2^{2x+1})$
$\frac{7}{1\cdot 4}=\frac{2^{2x+1}}{2^{x+5}}$
$5=2^{x-4}$
$\log_2 5=x-4$
$2\cdot 32=x-4$
$x=6\cdot 32$

27. $1\cdot 3\times 10^4\times e^{3t-1}=6\cdot 5\times 10^5\times e^{t+3}$
$\frac{e^{3t-1}}{e^{t+3}}=\frac{6\cdot 5\times 10^5}{1\cdot 3\times 10^4}$
$e^{2t-4}=50$
$\ln 50=2t-4$
$3\cdot 912=2t-4$
$2t=7\cdot 912$
$t=3\cdot 96$

28. $8\cdot 9(e^{4x+7})=32\cdot 5(e^{3x-1})$
$\frac{e^{4x+7}}{e^{3x-1}}=\frac{32\cdot 5}{8\cdot 9}$
$e^{x+8}=3\cdot 652$
$\ln 3\cdot 652=x+8$
$1\cdot 295=x+8$
$x=-6\cdot 70$

29. $3\cdot 2(2\cdot 56)^x=18\cdot 9(1\cdot 73)^{x+2}$
$\ln\left[3\cdot 2(2\cdot 56)^x\right]=\ln\left[18\cdot 9(1\cdot 73)^{x+2}\right]$
$\ln 3\cdot 2+\ln 2\cdot 56^x=\ln 18\cdot 9+\ln 1\cdot 73^{x+2}$
$\ln 3\cdot 2+x\ln 2\cdot 56$
$=\ln 18\cdot 9+(x+2)\ln 1\cdot 73$
$1\cdot 163+0\cdot 94x=2\cdot 939+0\cdot 548(x+2)$
$0\cdot 392x=2\cdot 872$
$x=7\cdot 33$

30. Let $y=2^x$. Then
$2^{2x}=(2^x)^2=y^2$
The equation is
$y^2-20y+64=0$
$(y-4)(y-16)=0$
$y=4$ or $y=16$
$2^x=2^2$ or $2^x=2^4$
$x=2$ or $x=4$

31. Let $y=3^x$. Then
$3^{x+2}=3^x.3^2=9y$
$3^{2x+5}=(3^x)^2.3^5=243y^2$
The equation is
$243y^2-36y+1=0$
$(27y-1)(9y-1)=0$
$27y-1=0$ or $9y-1=0$
$y=\frac{1}{27}$ or $y=\frac{1}{9}$
$3^x=3^{-3}$ or $3^x=3^{-2}$
$x=-3$ or $x=-2$

32. Let $y=3^x$. Then
$3^{2x+1}=3(3^x)^2=3y^2$
The equation is
$3y^2-28y+9=0$
$(3y-1)(y-9)=0$
$3y-1=0$ or $y-9=0$
$y=\frac{1}{3}$ or $y=9$
$3^x=3^{-1}$ or $3^x=3^2$
$x=-1$ or $x=2$.

33. Let $y=e^x$. Then the equation can be written:
$\frac{2}{y}=y-1$
$2=y^2-y$

$y^2 - y - 2 = 0$
$(y-2)(y+1) = 0$
$y = 2$ or $y = -1$

$e^x = 2$ or $e^x = -1$
(not possible)
$x = \ln 2$
$x = 0 \cdot 70$.