THE INSTITUTE OF EDUCATION

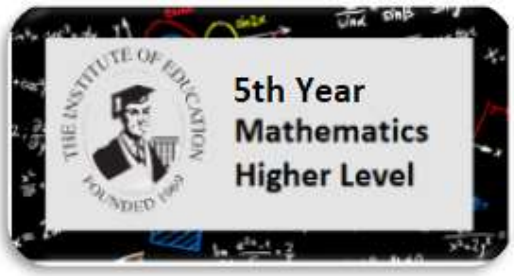
Maths

Louise Boylan

Higher Level

2020-21

Trigonometry – Solving Triangles



Date: _____

Student name: _____

5th Year Maths – Louise Boylan

UNIT 14 – TRIGONOMETRY II

Solving triangles

This unit of learning is divided into the following sections:

Section A – Sine Rule

Section B –Cosine Rule

Section C – Area of a Triangle

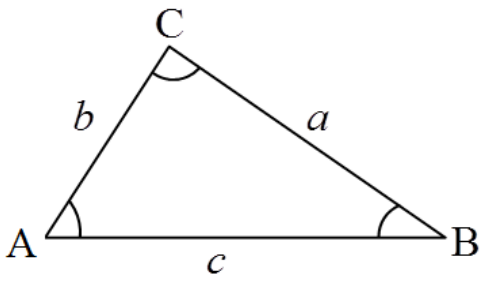
Section A – Sine Rule

The following formulae can be used on all types of triangles.

Sine rule:

If you are trying to find a missing side:

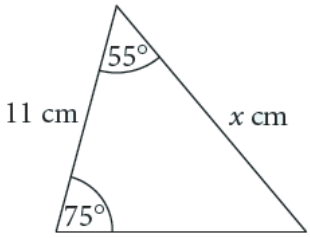
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$



If you are trying to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Example 1: Find the value of x.
Give your answer to one decimal place.

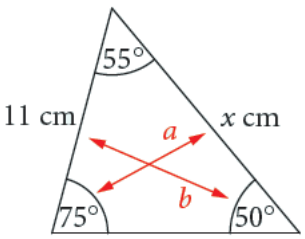


Solution:

We do not have two angle/side pairs.
Therefore, we need to find the third angle within the triangle:

Third angle =
180° − 55° − 75°
Third angle = 50°

Label one pair as a and one pair as b



$a = x, A = 75°, b = 11, B = 50°$

We are looking for a side, so we put the sides on top:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

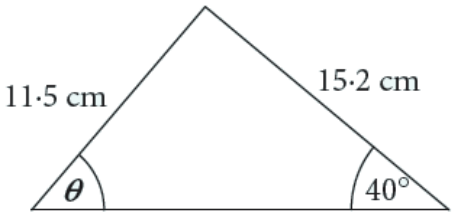
$$\frac{x}{\sin 75°} = \frac{11}{\sin 50°} \quad \text{(multiply both sides by sin 75°)}$$

$$x = \frac{11(\sin 75°)}{\sin 50°}$$

$$x = 13{\cdot}87019276$$

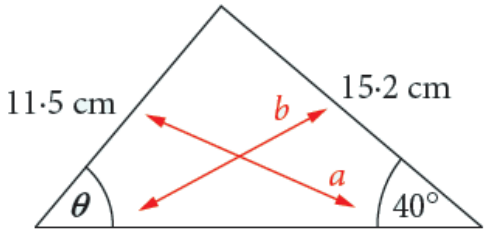
$$x = 13{\cdot}9 \text{ cm}$$

Example 2: Find the value of the acute angle, ϑ.
Give your answer in degrees and
minutes, correct to the nearest minute.



Solution:

Pick out the two angle/side pairs.
Label one pair as a and the other pair as b.
We are looking for an angle, so we put the angles on top:



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 40°}{11·5} = \frac{\sin \theta}{15·2}$$

$$\frac{15·2(\sin 40°)}{11·5} = \sin \theta$$

$$0·8495975363 = \sin \theta$$

$$\sin^{-1}(0·8495975363) = \theta$$

$$58·16792216 = \theta$$

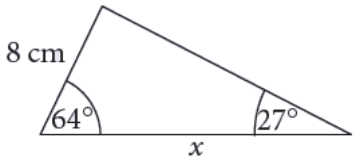
$$58° \; 10' \; 4·52'' = \theta$$

$$58° \; 10' = \theta$$

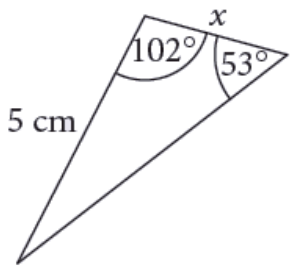
Practice Question A

1. Using the sine rule, find the side x in each of the following triangles.
Give your answers to one decimal place.

(i)



(ii)



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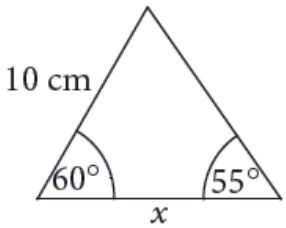
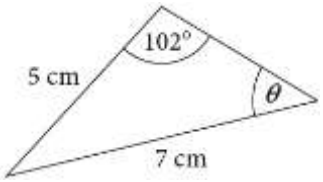
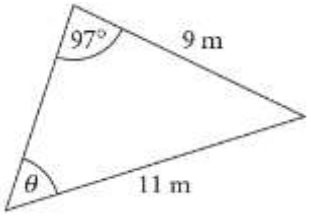


2. Use the sine rule to find the measure of the acute angle ϑ in each of the following triangles.
Give your answers to one decimal place.

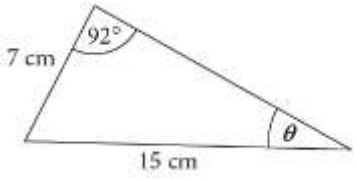
(i)



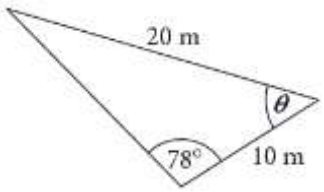
(ii)



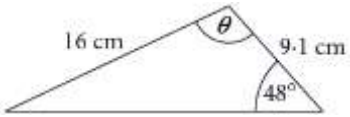
(iii)



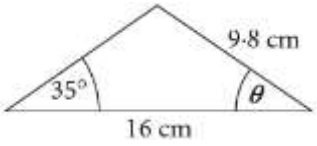
(iv)



(v)



(vi)



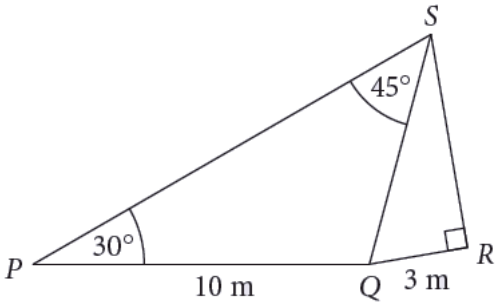
3. The diagram shows two triangles *PQS* and *QRS*.

Find, in surd form:

(i) $|SQ|$

(ii) $|SR|$

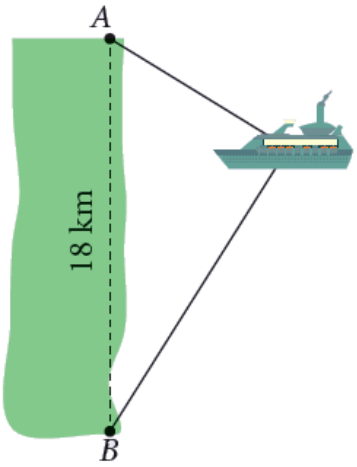
(iii) $|PS|$



4. A ship is anchored off a long straight shoreline that runs north to south.
From two observation points, *A* and *B*, 18 km apart on the shore, the bearings of the ship are N 30° E from point *B* and S 55° E from point *A*.

(i) What is the distance from the ship to each of the observation points?
Give your answer to one decimal place.

(ii) Find the shortest distance from the ship to the shoreline. Give your answer to the nearest metre.



5. Suraj and Graham want to measure the height of a vertical telephone mast. They stand on opposite sides of the mast, a distance 80 m apart. Suraj measures the angle of elevation to the top of the mast to be 57°. Graham measures the angle of elevation to the top of the mast to be 48°.

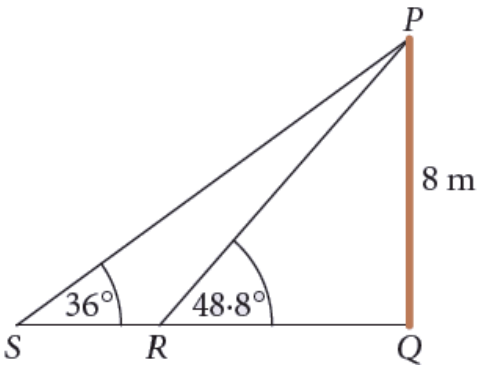
(i) Draw a mathematical model of the information given.

(ii) Find the height of the mast. Give your answer to the nearest metre.

6. The diagram shows a vertical telephone pole, [*PQ*], of height 8 m. Two wires [*PR*] and [*PS*] are connected from the top of the pole to the horizontal ground.

(i) the length of the wire, $|PR|$, correct to one decimal place

(ii) the distance between the two points *S* and *R*, where the wires are connected to the ground.
Give your answer to the nearest metre.



Section B – Cosine Rule

The following formula can be used on all types of triangles.

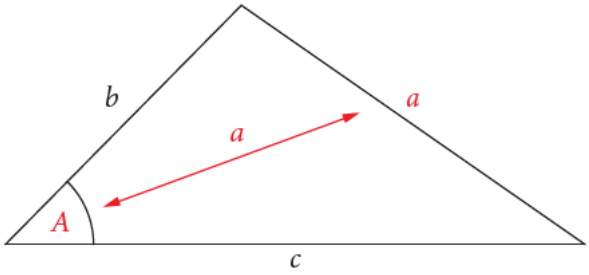
Cosine rule:

To be used when dealing with three sides and an angle.

Let the angle/side pair be the As.

Then,

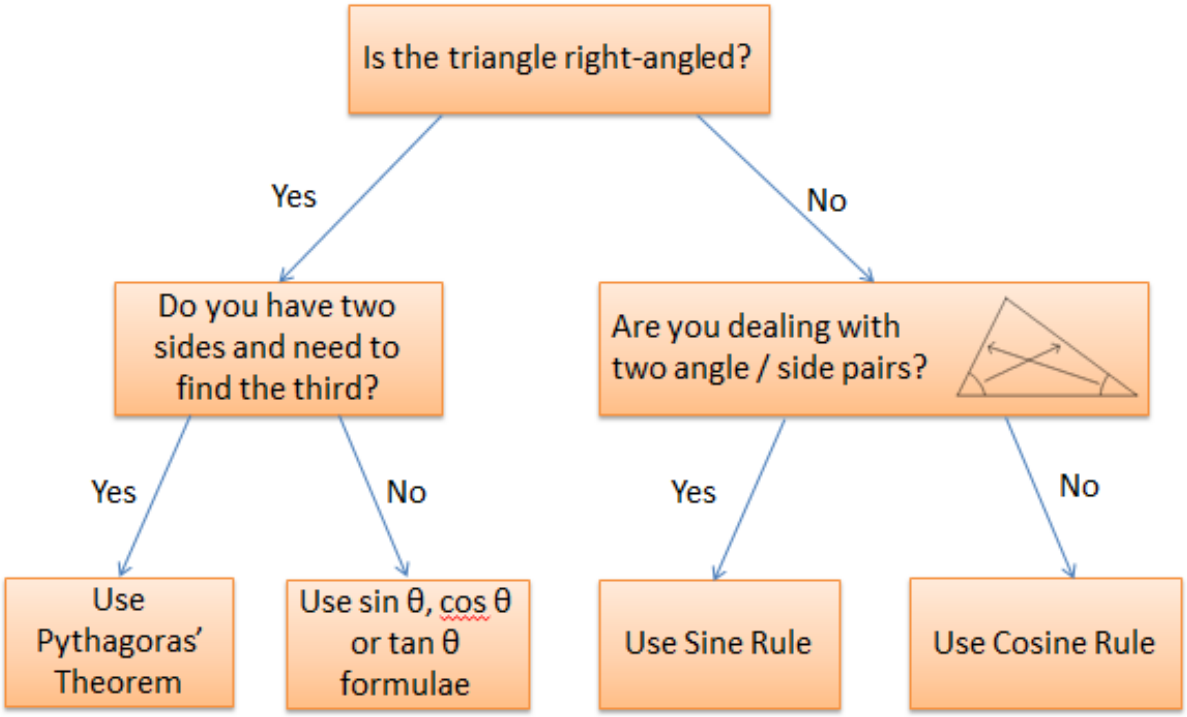
$$a^2 = b^2 + c^2 - 2bc\cos A$$



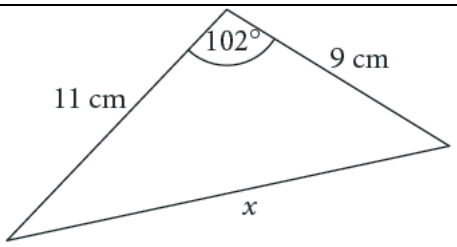
It can help to put a bracket around the last part:

$$a^2 = b^2 + c^2 - [2bc\cos A]$$

Flowchart for solving triangles:



Example 1: Find the value of x.
Give your answer to one decimal place.



Solution:

Label the angle/side pair as a.
Label one of the other sides b and the third side c.

$a = x, \quad b = 9, \quad c = 11, \quad A = 102°$

$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = (9)^2 + (11)^2 - [2(9)(11)\cos 102°]$

$x^2 = 81 + 121 - [198\cos 102°]$

$x^2 = 202 - [-41{\cdot}16651478]$

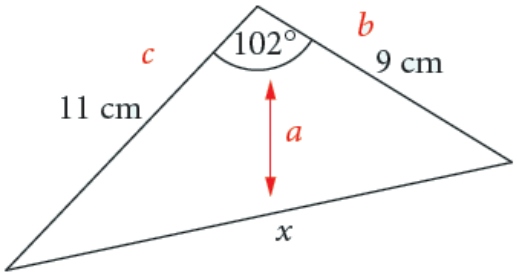
$x^2 = 202 + 41{\cdot}16651478$

$x^2 = 243{\cdot}16651478$

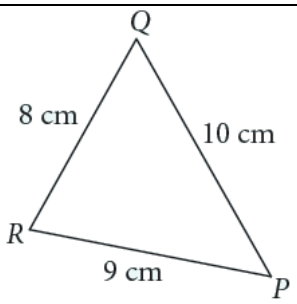
$x = \sqrt{243{\cdot}16651478}$

$x = 15{\cdot}593797$

$x = 15{\cdot}6 \text{ cm}$



Example 2: The diagram shows triangle PQR.
Find $|\angle PRQ|$
Give your answer to two decimal places.



Solution:

Label the angle/side pair as a.
Label one of the other sides b and the third side c.

$a = 10, \quad b = 8, \quad c = 9, \quad A = |\angle PRQ|$

$a^2 = b^2 + c^2 - 2bc\cos A$

$(10)^2 = (8)^2 + (9)^2 - [2(8)(9)\cos A]$

$100 = 64 + 81 - [144\cos A]$

$100 = 145 - [144\cos A]$

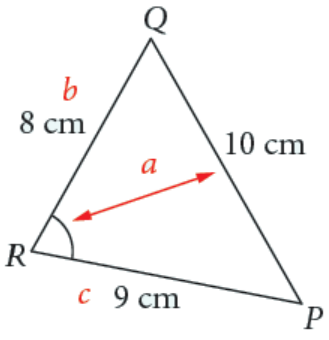
$[144\cos A] = 145 - 100 \qquad$ (rearrange)

$144\cos A = 45$

$\cos A = \frac{45}{144}$

$A = \cos^{-1}\left(\frac{45}{144}\right)$

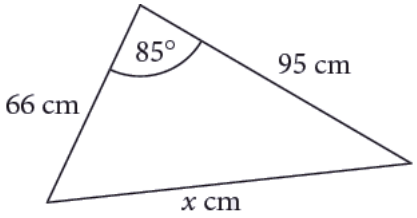
$A = 71{\cdot}79°$



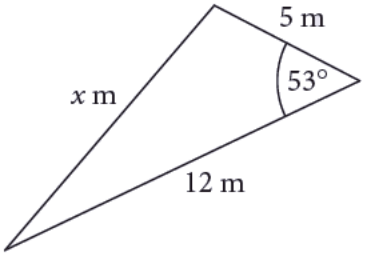
Practice Question B

1. Using the cosine rule, find the side x in each of the following triangles.
Give your answers to one decimal place.

(i)

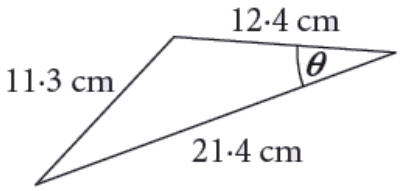


(ii)

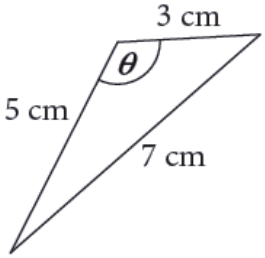


2. Find angle ϑ in each of the following triangles.
Give your answers in degrees and minutes, correct to the nearest minute.

(i)



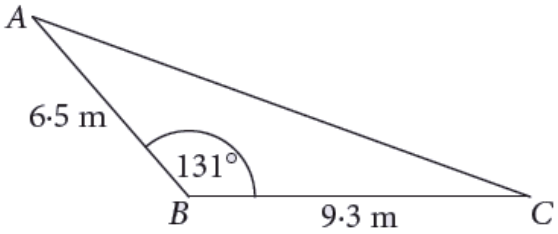
(ii)



3. The diagram shows the triangle ABC.
Find, correct to one decimal place:

(i) $|AC|$

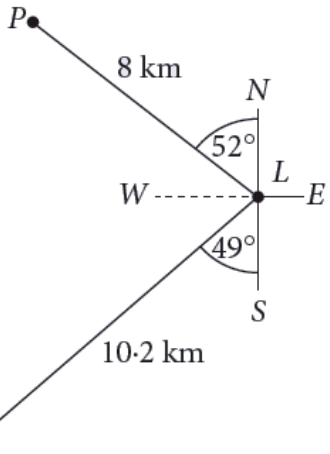
(ii) $|\angle BAC|$



4. From a lighthouse, L, a ship P is 8 km away on a bearing of N 52° W and ship Q is 10·2 km away on a bearing of S 49° W.
This is shown in the diagram.
Find:

(i) $|\angle PLQ|$

(ii) The distance between the two ships



5. From a lighthouse L, ship A is 4 km away on a bearing of N 35° E and ship B is 6 km away on a bearing of S 48° E for the lighthouse.

(i) Draw a mathematical model of the information given.

(ii) Find the distance between the two ships, correct to one decimal place.

6. Two adjacent sides of a parallelogram are of length 9·5 cm and 7 cm, and the shorter diagonal is of length 8 cm.

(i) Draw a diagram of the parallelogram, showing all known dimensions.

(ii) Find the angles of the parallelogram. Give your answer to the nearest degree.

(iii) Find the length of the other diagonal of the parallelogram.

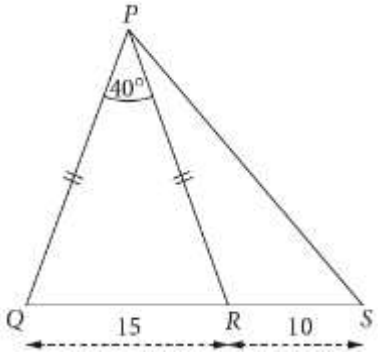
7. Two ships leave a port *P*, at the same time. *Princess Sophie*, *S*, sails on a bearing N 75° W at a speed of 16 km/hr, and *Princess Regina, R*, sails on a bearing of N 10° W.
After three hours the angle *PRS* is 80°.

(i) Draw a mathematical model to represent the information given.

(ii) Find the distance between the ships at this time, to one decimal place.

(iii) Find the speed of the *Princess Regina,* to one decimal place.

8. In the triangle *PQR*, $|PQ| = |PR|$, $|QR| = 15$ cm and $|\angle RPQ| = 40°$.
S is a point on *QR* such that $|RS| = 10$ cm.

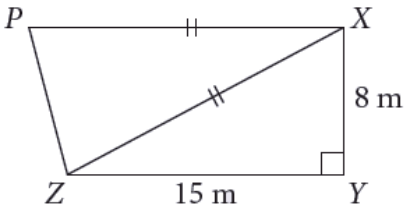


(i) Find $|PR|$, correct to the nearest centimetre.

(ii) Find $|PS|$, correct to the nearest centimetre.

9. Find, correct to one decimal place, the size of the smallest angle of the triangle which has sides of length 3, 5 and 7. Give your answer in degrees.

10. In the diagram, the triangle *XYZ* is right-angled.
XP is parallel to *ZY* and $|XP| = |XZ|$, as shown.
$|XY| = 8$ m and $|ZY| = 15$ m. Find:



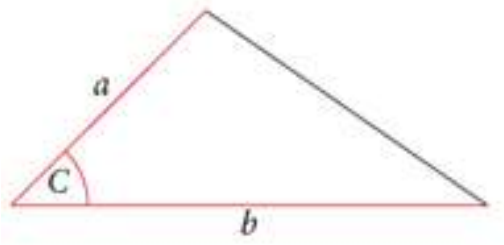
(i) $|XZ|$

(ii) $|PZ|$, correct to the nearest metre.

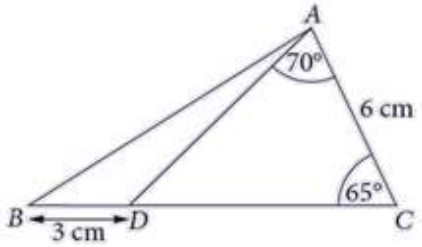
Section C – Area of a triangle

When dealing with a triangle, where we are given two sides, a and b, and measure of the angle in between these sides, we use the following formula:

$$\text{Area} = \frac{1}{2}ab\sin C$$



Example: ABC is a triangle and D is a point on $[BC]$, as shown. $|BD|$ = 3cm, $|AC|$ = 6 cm, $|\angle ACD| = 65^\circ$ and $|\angle DAC| = 70^\circ$.



Find:

(a) $|DC|$, correct to two decimal places.

(b) The area of the triangle ABC, correct to one decimal place.

Solution:

(a) Use Sine rule:

$$\frac{|DC|}{\sin|\angle DAC|} = \frac{|AC|}{\sin|\angle ADC|}$$

$$\frac{|DC|}{\sin 70^\circ} = \frac{6}{\sin 45^\circ}$$

$$|DC| = \frac{6\ \sin 70^\circ}{\sin 45^\circ}$$

$$|DC| = 7 \cdot 97 \text{ cm}$$

(b) Area of triangle ABC:

$$\frac{1}{2}|AC||BC|\sin|\angle ACB|$$

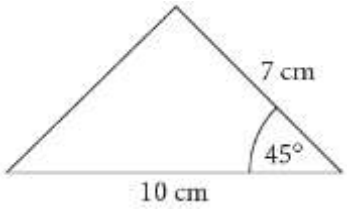
$$\frac{1}{2}\ (6)\ (10 \cdot 97)\ \sin 65^\circ$$

29·8 cm^2

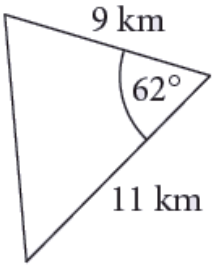
Practice Question C

1. Find the area of each of the following triangles. Give your answer in surd form, where possible. Otherwise give to one decimal place.

(i)



(ii)



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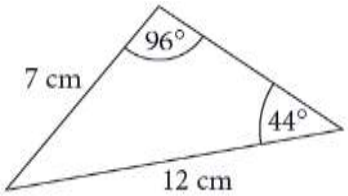

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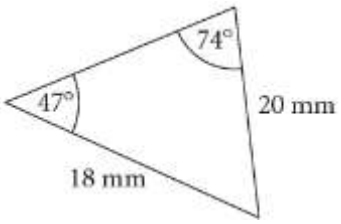
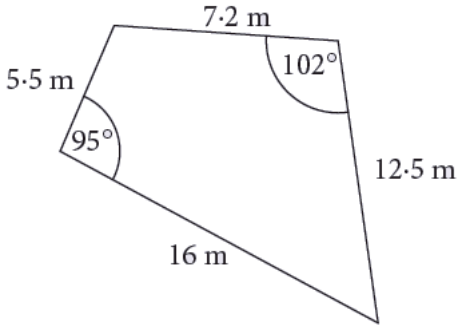
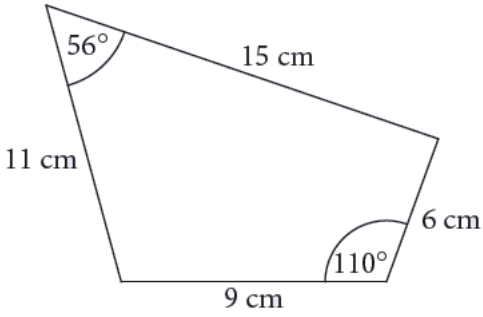


2. Find the area of each of the following quadrilaterals.
Give your answer correct to one decimal place.

(i)

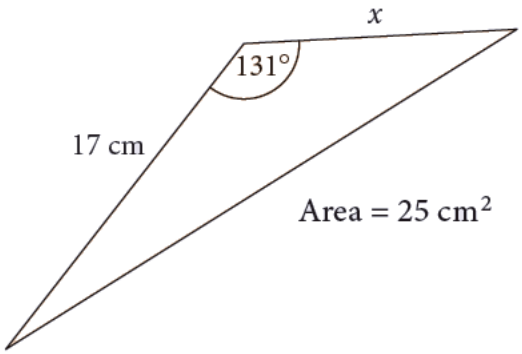


(ii)

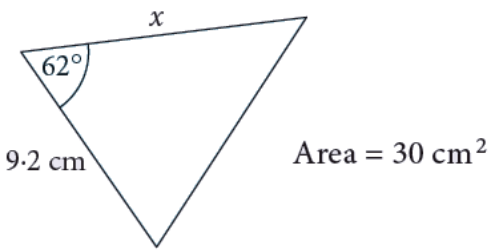


3. Find the value of *x,* in each case, given the area of the triangle shown.
Give your answer to one decimal places.

(i)



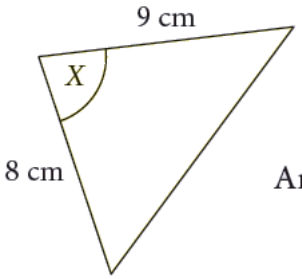
(ii)



4. The area of an equilateral triangle is $4\sqrt{3}$ cm^2.
Find the length of a side of the triangle.

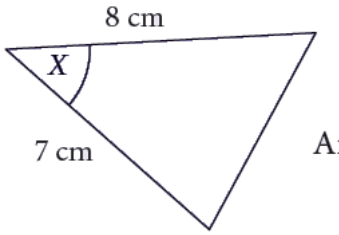
5. Find the measure of the acute angle X in each case, given the area of the triangle shown.
Give your answer to two decimal places.

(i)



Area = 30 cm^2

(ii)



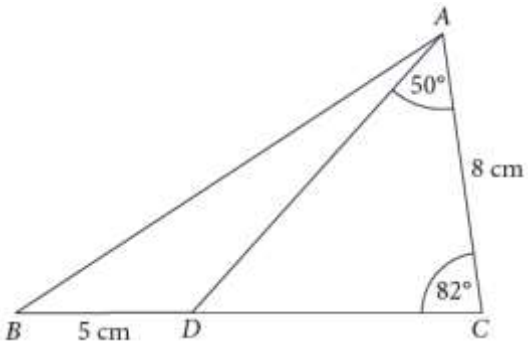
Area = 23·5 cm^2

6. In the triangle ABC, D is a point on $[BC]$, as shown.
$|BD| = 5$ cm, $|AC| = 8$ cm, $|\angle DCA| = 82°$
and $|\angle CAD| = 50°$. Find

(i) $|DC|$, correct to two decimal places.

(ii) $|AB|$, correct to one decimal place.

(iii) the area of triangle ABC, to one decimal place.

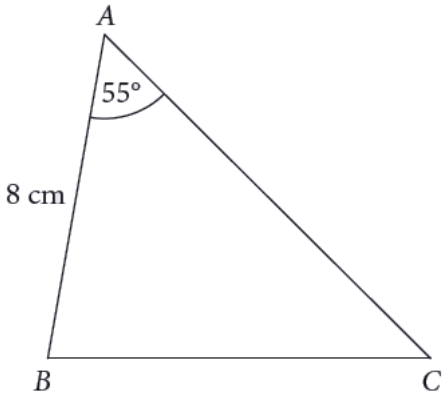


7. The area of the triangle ABC is 33 cm^2.
$|AB| = 8$ cm and $|\angle CAB| = 55°$.
Find:

(i) $|AC|$, correct to one decimal place

(ii) $|BC|$, correct to one decimal place

(iii) $|\angle ABC|$, correct to the nearest degree.



Answers

Practice Questions A

1. **(i)** 17·6 cm **(ii)** 2·6 cm **(iii)** 11·1 cm

2. **(i)** 44·3˚ **(ii)** 54·3˚ **(iii)** 27·8˚
(iv) 72·7˚ **(v)** 107˚ **(vi)** 75·5˚

3. **(i)** $5\sqrt{2}$ m **(ii)** $\sqrt{41}$ m **(iii)** $5 + 5\sqrt{3}$ m

4. **(i)** 9 km, 14·8 km **(ii)** 7,372 m

5. **(i)** - **(ii)** 95 m

6. **(i)** 10·6 m **(ii)** 4 m

Practice Questions B

1. **(i)** 111·9 cm **(ii)** 9·8 m

2. **(i)** 24˚ 13’ **(ii)** 120˚

3. **(i)** 14·4 m **(ii)** 29·4˚

4. **(i)** 79˚ **(ii)** 11·7 km

5. **(i)** - **(ii)** 7·6 km

6. **(i)** - **(ii)** 124˚ **(iii)** 14·6 cm

7. **(i)** - **(ii)** 44·2 km **(iii)** 9·3 km/h

8. **(i)** 22 cm **(ii)** 27 cm

9. 21·8˚

10. **(i)** 17 m **(ii)** 8 m

Practice Questions C

1. **(i)** $\frac{35\sqrt{2}}{2}$ cm^2 **(ii)** 43·7 km^2 **(iii)** 27 cm^2 **(iv)** 154 mm^2

2. **(i)** 87·8 m^2 **(ii)** 93·8 cm^2

3. **(i)** 3·9 cm **(ii)** 7·4 cm

4. 4 cm

5. **(i)** 56·44˚ **(ii)** 57·06˚

6. **(i)** 8·25 cm **(ii)** 10·7 cm **(iii)** 52·5 cm^2

7. **(i)** 10·1 cm **(ii)** 8·6 cm **(iii)** 74˚