Chapter 3 Resonance and Stretched Strings

Fundamental mode of vibration:

- If a string is fixed at both ends and plucked in the centre the most simple or fundamental type of vibration is shown in the above diagram.
- For the fundamental mode the string vibrates at its lowest frequency

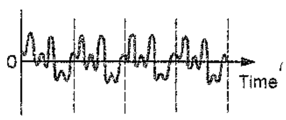
Harmonics: *2nd harmonic* *3rd harmonic*

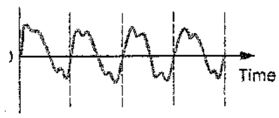
- **Harmonics are multiples of the fundamental mode of vibration**
- The diagram above shows the string vibrating in the second and the third harmonic
- The **overtones** are all the harmonics except the first harmonic.

Pitch:

- **Pitch is related to the frequency of a sound**. The higher the frequency the higher the pitch
- "pitch is to sound as colour is to light"

Quality: *violin* *piano*





- The same note played on two different instruments does not sound the same e.g. the piano and the violin.
- **The quality of a sound depends on the number and intensity of the harmonics (overtones) present**

Loudness:

- **The loudness of a sound depends on the amplitude of the vibration of the sound wave**.
- Loudness is subjective.

Characteristics of a sound:

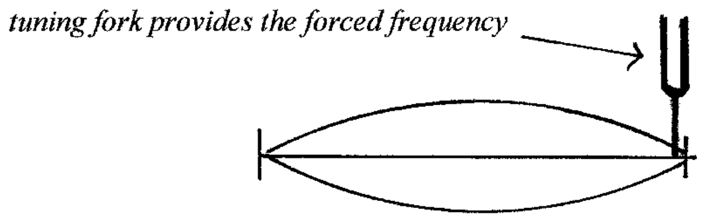
- The characteristics of a sound are pitch, quality and loudness. These are the three ways in which sounds differ.

Natural frequency:

- When a system that is capable of vibrating is made to vibrate it will do so at its natural frequency.
- When a stretched elastic band is plucked in the middle it will vibrate at its natural frequency

Forced frequency:

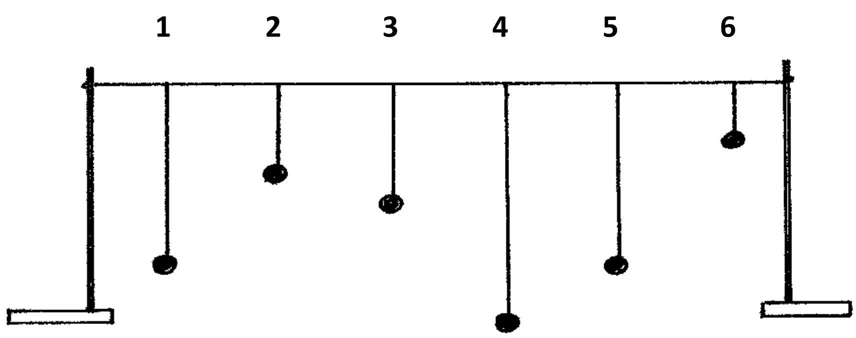
- When an external vibration force acts on a system that is capable of vibrating the external force provides the forced frequency.
- Touch a vibrating tuning fork to a stretched string. The string vibrates at the same frequency as the tuning fork. The frequency of the tuning fork is the forced frequency.



Resonance is the transfer of energy between two bodies of the same natural frequency

- Resonance happens when a vibrating system responds with maximum amplitude to a forced frequency.
- An example would be a singer who shatters a wine glass.
  - → The wine glass is the system capable of vibrating. It has a natural frequency.
  - → The sound from the singer provides the forced frequency.
  - → When forced frequency equals the natural frequency resonance happens. The glass shatters.

Demonstration (laboratory) of resonance: **(Barton's pendulums)**



- A number of pendulums are arranged as shown above.
- Pendulum 1 is made swing in and out of the plane of the page.
- All the pendulums start to swing a little but pendulum 5 swings most.
- Pendulums 1 and 5 have the same length and therefore the same natural frequency.
- Energy is transferred back and forth between the pendulums of the same natural frequency.

Note: $T = 2\pi\sqrt{\frac{l}{g}}$ $T = \frac{1}{f}$ for a pendulum.

Factors affecting the natural frequency of a stretched string:

The natural frequency of a stretched string depends on

- The length of the string
- The tension force the string is subjected to.
- The mass per unit length of the string. (mass of 1 m of string)

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$ ………………………………………. P. 59

(f = frequency; l = length of string; T = tension; μ = mass per unit length)

Note: The above formula assumes that the string vibrates at the fundamental frequency. For the higher harmonics the formula is written as

$$f = \frac{n}{2\,l}\sqrt{\frac{T}{\mu}}$$

(n = number of the harmonic)

Sample question 1: Calculate the fundamental frequency of a string 80 cm long, subjected to a tension of 22 N if the mass per unit length is $4\times10^{-4}\,kg\,m^{-1}$

Solution:

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

$$f = \frac{1}{2\times0.8}\sqrt{\frac{22}{4\times10^{-4}}}$$

$$f = 146.6Hz$$

Sample question 2: A string 70 cm long has a fundamental frequency of 256 Hz when subjected to a tension of 45 N. Calculate the mass per unit length of the string.

Solution:

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

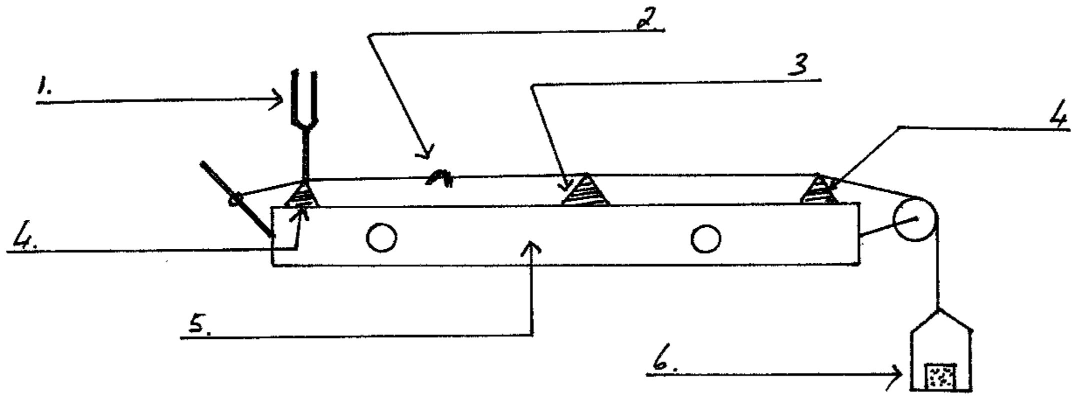
Now square both sides $$f^2 = \frac{1}{4\,l^2}\times\frac{T}{\mu}$$

Rearrange to get $$\mu = \frac{1}{4\,l^2}\times\frac{T}{f^2}$$

$$\mu = \frac{1}{4\times(0.7)^2}\times\frac{45}{(256)^2}$$

$$\mu = 3.5\times10^{-4}\,kg\,m^{-1}$$

Mandatory Experiment: To investigate the variation of fundamental frequency of a stretched string with length:



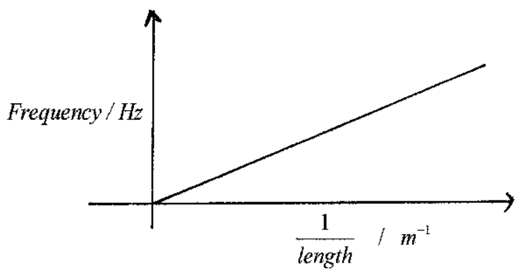
- Subject the string to a fixed tension and keep this tension constant.
- Place a small paper rider on the string.

Data:

- Note and record the frequency of the first tuning fork. Now let the base of this vibrating tuning fork touch the string
- Adjust the moveable bridge until the string gives resonance in the fundamental frequency, i.e. the paper rider is thrown off the string.
- Note and record the length of the string for this first position of resonance.

Calculations:

- Note and record several values of tuning fork frequencies and the corresponding lengths of string when resonance happens.
- Plot a graph of frequency on the y-axis and the **reciprocal of length** on the x-axis.
- A straight line graph through the origin verifies that **frequency is inversely proportional the length.** ($f \propto \frac{1}{l}$)



Accuracy:

- Avoid the error of parallax when measuring the length of string with the metre stick
- Avoid tuning forks of very high frequency. Such tuning forks cause resonance for very small lengths of string. Measuring small lengths results in greater percentage errors.
- Always ensure the string resonates in the fundamental mode. The paper rider falls off the centre of the string due to an antinode at the centre of the string and nodes at the bridges.

Sample question 3: A student investigated the variation of the fundamental frequency f of a stretched string with its length l and obtained the following data

l / m	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f / Hz	675	455	335	273	230	193	173

(i) Why was the tension in the string kept constant during the investigation?

(ii) Using the above data draw a suitable graph on graph paper to show the relationship between the fundamental frequency of the stretched string and its length.

(iii) From the graph estimate the length of the string when its fundamental frequency has a value of 256 Hz

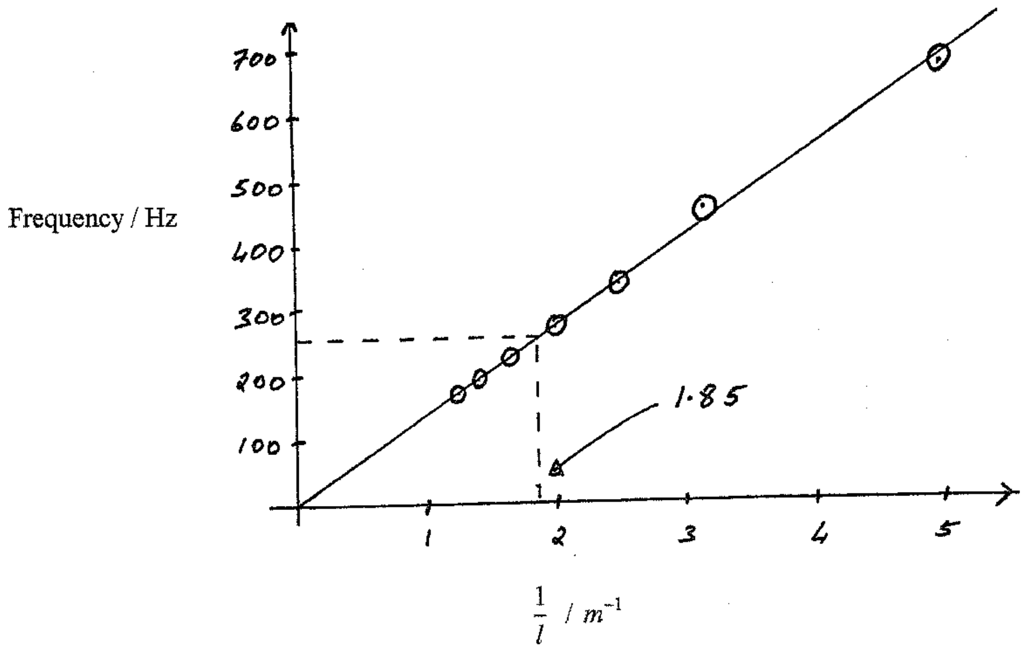
Solution:

(i) Frequency is dependent on length and tension. To investigate one of these variables, in this case length, the other variable, tension, must be constant

(ii) The **data has to be adjusted**

- Get the **reciprocal of the values of length**
- Leave the frequency as it is

f / Hz	675	455	335	273	230	193	173
$\frac{1}{l}$ / m^{-1}	5	3.33	2.5	2	1.67	1.43	1.25



- A straight line graph through the origin verifies that **frequency is inversely proportional the length.** ($f \propto \frac{1}{l}$)

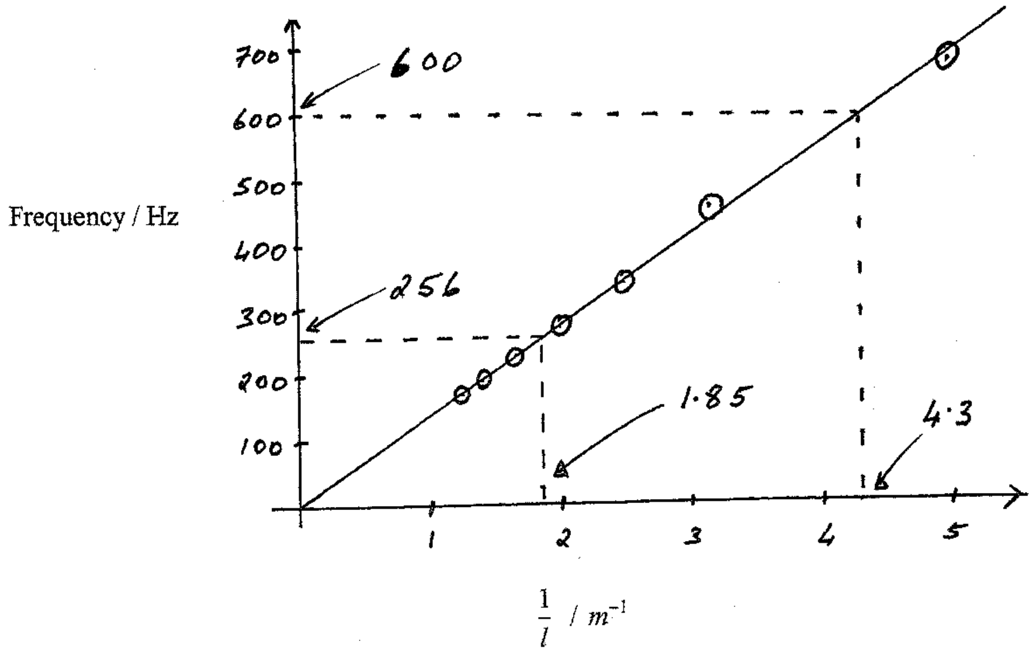
(iii) when the frequency is 256 Hz the value of the reciprocal length is 1.85. The value of length is given by $\frac{1}{1.85}$ = 0.54 m

Sample question 4: A student investigated the variation of the fundamental frequency f of a stretched string with its length l and obtained the following data.
The string was under a constant tension of 50 N.
By plotting a suitable graph on graph paper calculate the mass per unit length of the string.

l / m	0.2	0.3	0.4	0.5	0.6	0.7	0.8
f / Hz	675	455	335	273	230	193	173

Solution: The graph required will be frequency on the y-axis and reciprocal length on the x-axis.

f / Hz	675	455	335	273	230	193	173
$\frac{1}{l}$ / m^{-1}	5	3.33	2.5	2	1.67	1.43	1.25



Step 1: slope of graph $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{600 - 256}{4.3 - 1.85} = 140.4$

Step 2: slope of graph $= \dfrac{f}{\frac{1}{l}} = f \times l$ therefore $f \times l = 140.4$

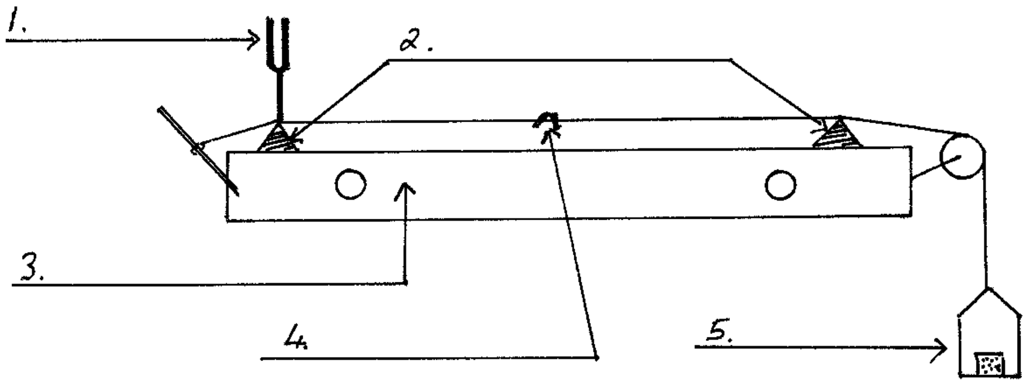
Step 3: square both sides of $f = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$ to get $f^2 = \dfrac{T}{4\,l^2\,\mu}$

$$\mu = \frac{T}{4\,l^2\,f^2}$$

$$\mu = \frac{50}{4 \times (140.4)^2}$$

$$\mu = 6.34 \times 10^{-4}\,kg\,m^{-1}$$

Mandatory Experiment: To investigate the variation of fundamental frequency of a stretched string with tension:



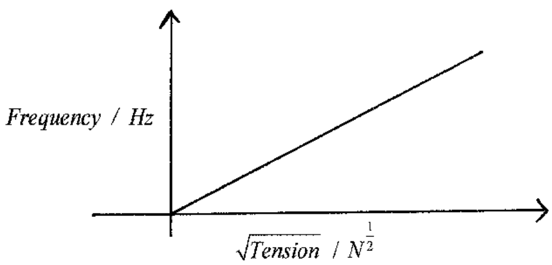
- Select a suitable length of string and keep this length constant
- Place a small paper rider on the string

Data:

- Note and record the frequency of the first tuning fork. Now let the base of this vibrating tuning fork touch the string.
- Adjust the weight in the pan (adjust the tension) until the string gives resonance in the fundamental frequency, i.e. the paper rider is thrown from the string.
- Note and record the tension for this first position of resonance.

Calculations:

- Note and record several values of tuning fork frequencies and the corresponding values of tension that caused resonance.
- Plot a graph of frequency on the y-axis and **square root of tension** on the x-axis.
- A straight line graph through the origin verifies that **frequency is proportional to the square root of the tension.** **(** $f \propto \sqrt{T}$)



Accuracy:

- The tension is equal to the weight of the pan and its contents.
- Avoid small values of frequency as they correspond to small values of weight. These small values lead to greater percentage errors.
- Always ensure the string resonates in the fundamental mode. The paper rider falls off the centre of the string due to an antinode at the centre of the string and nodes at the bridges.

Sample question 5: The relationship between the natural frequency of a stretched string and its tension was investigated by a student. The length of the string was 64 cm. The following values of frequency f and tension T were obtained.

f / Hz	256	288	320	384	427	480	512
T / N	5	6	9	12	14	18	22

By drawing a suitable graph on graph paper calculate

(i) the frequency when the tension was 16 N

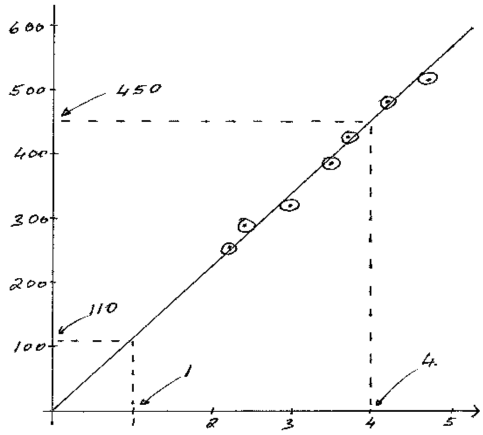
(ii) the mass per unit length of the string

Solution: the data has to be adjusted

- get the **square root of all the tension values**
- leave the frequency values as they are

f / Hz	256	288	320	384	427	480	512
$\sqrt{T}$ / $N^{\frac{1}{2}}$	2.2	2.4	3	3.5	3.7	4.2	4.7

Frequency / Hz



$\sqrt{tension}$ / $\sqrt{N}$

(i) When the tension is 16 N the square root of tension is 4. Now 4 on the x-axis corresponds to 450 Hz on the y-axis.

(ii)

Step 1: slope of graph $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{450 - 110}{4 - 1} = 113.3$

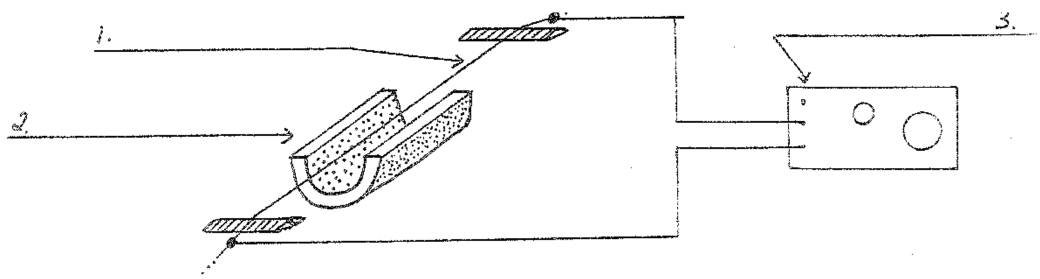
Step 2: slope of graph $= \dfrac{f}{\sqrt{T}}$ therefore $\dfrac{f}{\sqrt{T}} = 113.3$

Step 3: square both sides of $f = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$ to get $f^2 = \dfrac{T}{4\,l^2\,\mu}$

Rearrange to get $\mu = \dfrac{T}{4\,l^2\,f^2} = \dfrac{1}{4\,l^2} \times \left\{\dfrac{\sqrt{T}}{f}\right\}^2$

$$\mu = \frac{1}{4 \times 0.64^2} \times \left\{\frac{1}{113.3}\right\}^2 = 4.75 \times 10^{-5}\ kg\,m^{-1}$$

Experiments on stretched strings: In the previous two experiments the forced frequency was provided by the vibrating tuning fork. An alternative way of providing a forced frequency is to use a magnet and a signal generator. The string would have to be an electrical conductor.



- The string made of conducting wire is placed between the poles of a strong U-shaped magnet.
- The wire is connected to a signal generator.
- The signal generator provides an a.c. signal at any desired frequency.
- A current flowing in a wire in a magnetic field will experience a force (the principle of the electric motor). The wire is forced to move.
- If the current is a.c. then the movement will be a vibration. The frequency of the vibration will be the frequency of the a.c.
- The signal generator is now providing the forced vibration instead of the tuning fork.
- Adjust the frequency from the signal generator until resonance happens.
- Now the resonance frequency , the length or the tension of the wire can be measured.