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Algebra Simultaneous Equations

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Solving 3 X Simultaneous Equations

- Remove fractions if present
- Number all 3 equations 1, 2 and 3
- Pick 2 of the 3 equations
- Choose 1 variable
- In both equations ensure that the chosen variable has the same coefficient with opposite signs in each equation
- Add the 2 equations
- The resulting equation is number 4
- Repeat from the beginning, choosing 2 different equations but the same variable
- Now you have equation 5
- Treat equations 4 and 5 as in previous studies
- Find one variable
- Then a second using equation 4 or 5
- Then the last variable using equation 1, 2 or 3
- Hint: if a variable is initially missing from one of your 3 equations then choose that variable to work with

$$a + b + c = 2$$
$$2a + 3b + c = 7$$
$$\frac{a}{2} - \frac{b}{6} + \frac{c}{3} = \frac{2}{3}$$

Solve the equation above:

2022 Paper 1 Q2

(ii) The areas of the three regions **K**, **L**, and **N** give the following three equations (including the equation from **part (b)(i)**):

$$4a + 3b + 3c = 807$$
$$28a + 9b + 3c = 879$$
$$76a + 15b + 3c = 663$$

Solve these equations to find the values of a, b, and c.

2013 Paper 1 Question 2

(b) Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

2018 Paper 1 Q 2

(a) Solve the simultaneous equations.

$$2x + 3y - z = -4$$
$$3x + 2y + 2z = 14$$
$$x - 3z = -13$$

Algebraically

- If we wish to solve for the point/s of intersection without graphing, then we use an algebraic approach

5 Steps to follow:
1. Look at the linear of the 2 equations (no square) and isolate one of the variables (Letters)
2. Substitute your now isolated expression into the non linear equation
3. Expand and simplify
4. Solve for your variable
5. Substitute in order to find the solutions for the second variable

Relevant again in the Circle when we look for a line intersecting a circle

$$xy = 4$$
$$2x - y + 2 = 0$$

(a) Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$
$$a + 2b + 1 = 0$$

2016 paper 1 Q2

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

Practical Trigonometry

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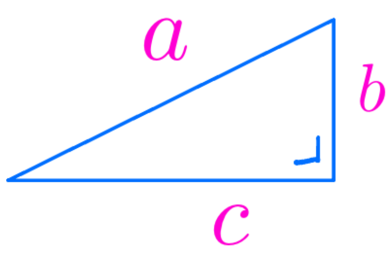
Pythagoras:

Helps you find the side in a right angled triangle

You typically need to know 2 sides to find the third



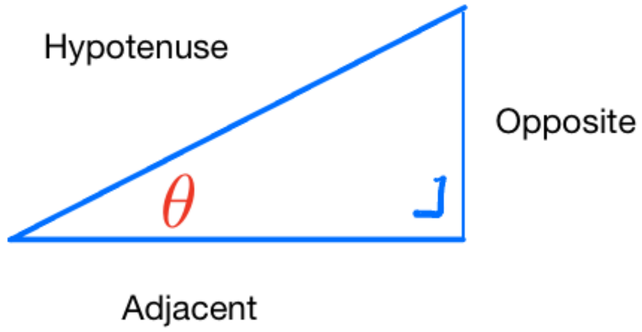
$$a^2 = b^2 + c^2$$

The Hypotenuse squared is equal to the sum of the squares of the remaining 2 sides

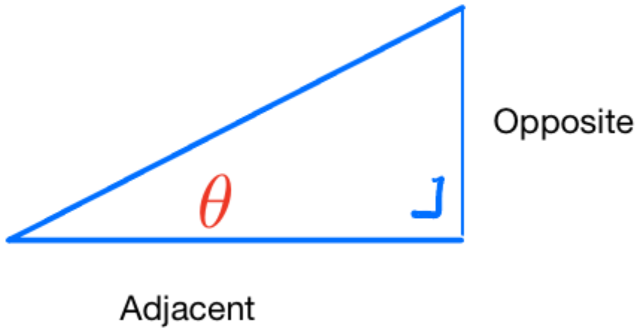
- It only works with right angled triangles
- The longest side is the one opposite the right angle and is known as the hypotenuse
- The remaining 2 sides have, up until now, been referred to as the height and base
- When we have 3 integers in the place of a, b and c we have a Pythagorean triple

Soh Cah Toa

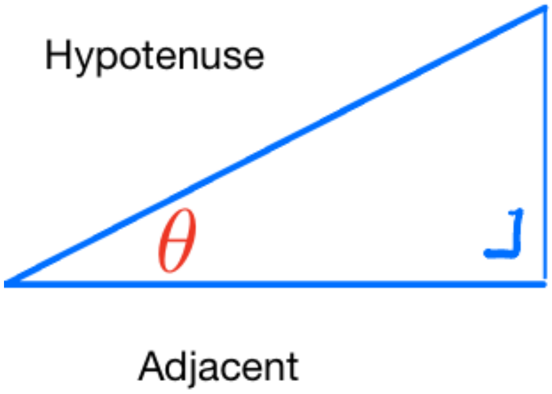
θ = the Greek letter Theta. It is often used to represent an unknown angle



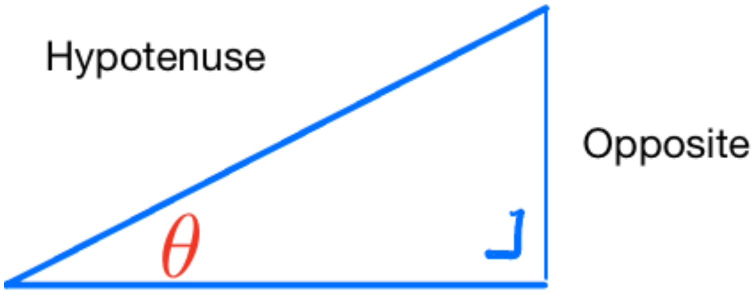
- Now lets start pairing up the sides
- There are 3 possible combinations



- When we have the opposite/adjacent we call this situation the Tangent (tan) of the angle
- We have 2 sides as a fraction thus creating a ratio



- When we have the adjacent/ hypotenuse we have the Cosine (Cos) of the angle



- Lastly we have the Opposite/ hypotenuse which is called the Sine (Sin) of the angle

- These 3 ratios are the building blocks of trigonometry and cover all possible combinations in right angle triangles
- Once you have labelled the sides correctly you will know which ratio to use and then be able to convert your ratio into an angle

$\sin\theta$ =	Opp/Hyp	Soh
$\cos\theta$ =	Adj/Hyp	Cah
$\tan\theta$ =	Opp/Adj	Toa

<u>Soh Cah Toa</u>

Used to find missing sides or angles in right angled triangles

Area of a Triangle - Trigonometric formula

- Finding the area of a triangle is nothing new
- We have used half x base x height with right angled triangles
- We've also seen a formula in co-ordinate geometry of the line where we have 3 points and carry out a translation
- Now we look at having 2 sides of a non-right angled triangle (it will also work for right angled triangles) with the angle in the middle of those 2 sides
- For this to work we use the formula:

$$Area = \frac{1}{2}ab\sin C$$

Tables book page 16

- The small letters represent **sides**
- The capital letters represent **angles**
- Answers should be in square units

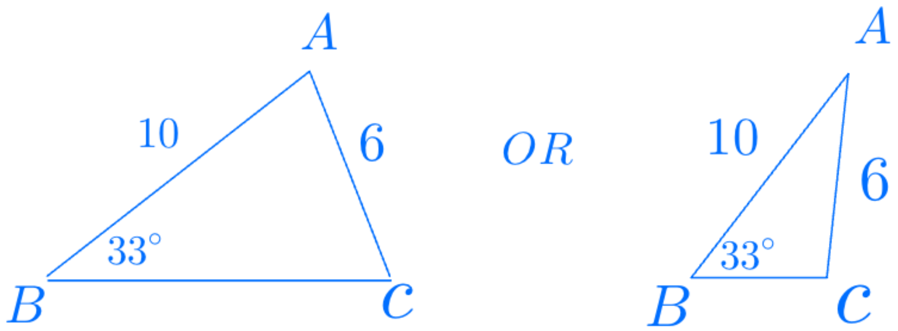
Sine Rule

- The Sine rule allows us to identify sides or angles in right angled and non right angled Triangles
- It allows us to do so by comparing opposite sides and angles
- You must know one side and it's opposite angle with one other side/angle for the sine rule to work
- If you do not have the necessary components then Cosine rule in the next section will typically be the answer
- Don't forget the area of a triangle above can help find missing sides and angles

$$\frac{a}{\sin A} = \frac{b}{sinB} = \frac{c}{\sin C}$$

Tables book page 16

- a, b and c refer to sides of a triangle
- A, B and C refer to the angles opposite the matching sides
- Can be used for right angled or non right angled triangles
- You need only look at 2 sides/angles at a time
- **When you are 2 sides and an angle not in between those sides you need to look for the ambiguous case.**



- Relevant when the diagram is not given
- You need to use the reference angles for sine to find the second possible angle

- Before we use Sine rule for problem solving I want to take a look at 2 formal proofs in Trigonometry
- A formal proof can be asked as a proof in the Leaving Certificate Exam

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

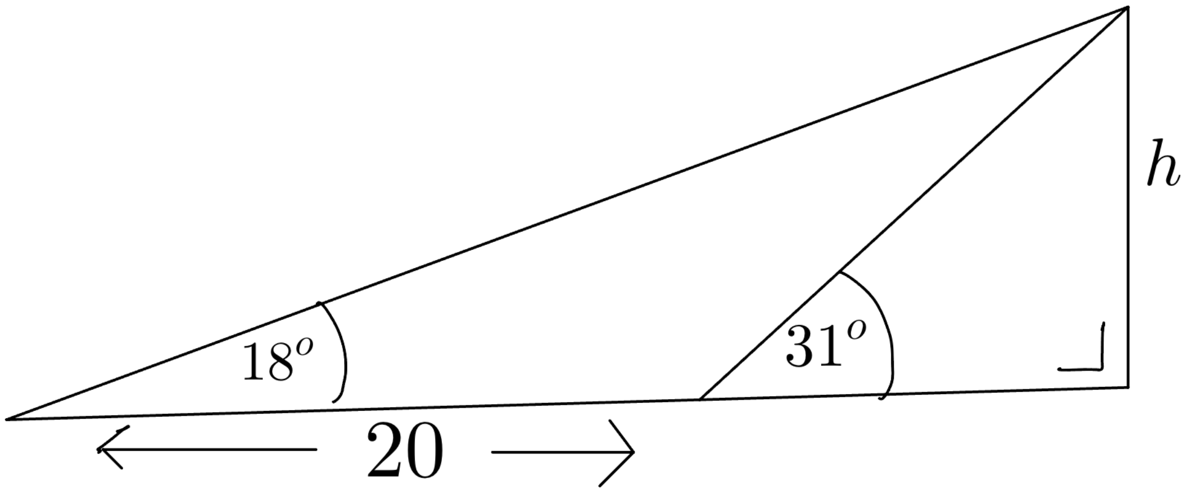
Tables book pg 15

- Again the small letters represent **sides**
- The capital letters represent **angles**
- Can be used in right angled and non right angled triangles
- Can be used when you have 3 sides and no angle

- The goal in practical trigonometry is to use any of the above rules when problem solving
- Try and look at the problem and then ask your yourself:

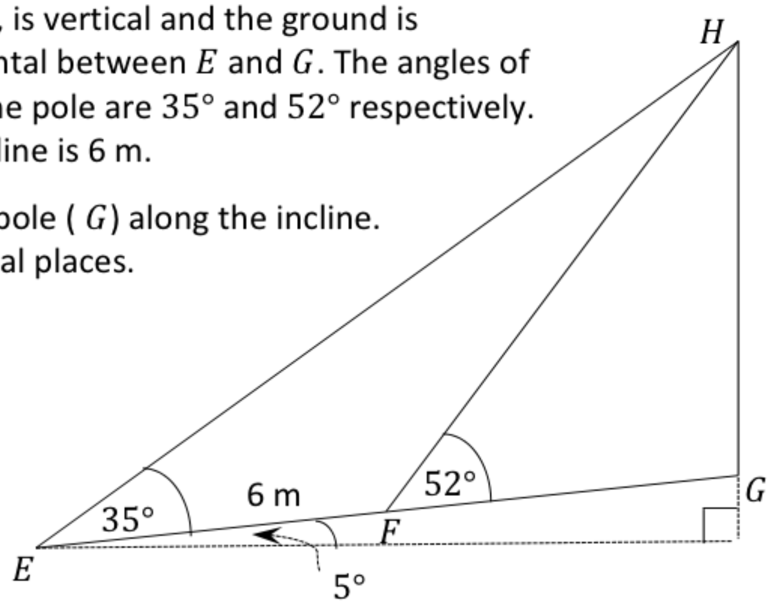
1. Are there any common sides?
2. Which triangle do I know the most about?
3. Are there any simple calculations I can do?
4. Which rule should I use to find the common side?
5. How can I find any remaining sides or angles in the question?

Find h correct to 1 decimal place



(a) A flagpole $[GH]$, shown in the diagram, is vertical and the ground is inclined at an angle of $5°$ to the horizontal between E and G. The angles of elevation from E and F to the top of the pole are $35°$ and $52°$ respectively. The distance from E to F along the incline is 6 m.

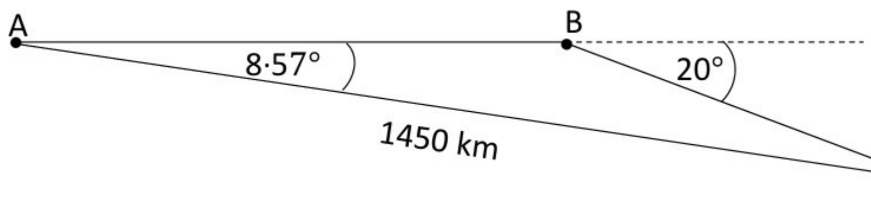
Find how far F is from the base of the pole (G) along the incline. Give your answer correct to two decimal places.



2021 Paper 2 Q9

(a) An aeroplane flies east from point A for 2 hours at a constant speed of 420 km per hour until it reaches point B. It then changes direction by heading 20° towards the south at the same speed until it reaches point C, as shown in the diagram below.
The direct distance from A to C is 1450 km and $|\angle BAC| = 8{\cdot}57°$.

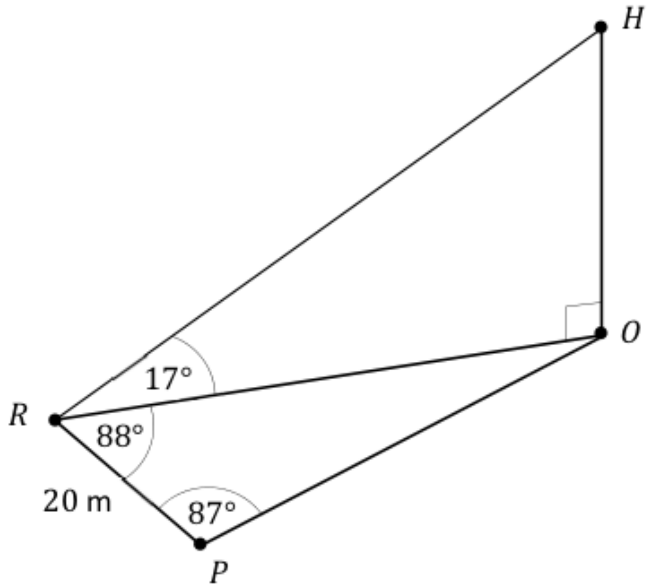
(i) Find how long it took to fly from B to C.
Give your answer correct to the nearest minute.



(b) Olga wants to measure the vertical height of a hill. The point H is at the top of the hill. The points R and P are 20 m apart on horizontal ground, at the bottom of the hill.

Olga measures the angle of elevation from R to H.
Taking O to be the point directly below H that is horizontal with R and P, Olga also measures the angles $\angle OPR$ and $\angle ORP$.
All of these are shown in the diagram below (not to scale).

Source: www.bikeforums.net/road-cycling



Work out the distance $|OH|$, the vertical height of the top of the hill relative to the points R and P. Give your answer correct to the nearest metre.

Co-ordinate Geometry of the Line

Midpoint and Distance

- The midpoint is about finding the point in between (in the middle of) 2 other points
- This is done by finding the average of the 2 X values and the average of the 2 Y values
- Another phrase can be to find the point that Bisects a line
- Bisect means to cut in half, so the midpoint is involved

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Found on page 18 of the Tables Book

- For Distance or length of a line segment we can use the formula below
- Another approach can be to use The Theorem of Pythagoras

$$Length/Distace = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Found on page 18 of the Tables Book

Slope of a Line

- Another word for slope is gradient
- Here we can use our formula given in the tables book
- We can also look at the slope as rise over run
- This can be identified by making a right angled triangle out of our 2 points
- The height is the rise and the base is the run
- If we get a positive slope that means our line is increasing
- If we get a negative slope then our line is decreasing
- The symbol for slope is 'm'

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} / \frac{rise}{run}$$

Found on page 18 of the Tables Book

$A||B$ This is how you say 2 lines are parallel with symbols

$A \perp B$ This is the symbol we use to show 2 lines are perpendicular

- Lines can be parallel or perpendicular
- Parallel lines have the exact same slope
- Perpendicular lines are at 90 degrees to each other and their slopes multiply to always give -1
- m1 x m2 = -1 when 2 lines are perpendicular
- This can be used to find a perpendicular slope if you already know one slope
- Alternatively invert the slope you have and change it's sign

Translations and area of a Triangle

- We have used translations before in the functions handout, when used graphically
- Now we will use it with points
- This follows on nicely from finding the length and slope of a line
- This means moving an object, in this case one or more points

- Now we can use our ability to translate to work out the area of a Triangle
- This is a useful formula as it relies on using 3 points
- Other formulas will use lengths (1/2 x base x height) or lengths and angles (Trigonometry)
- One of our points must be (0,0) for this formula to work, therefore translation is often required
- Since we are finding area our answer must be positive and given in the correct units squared
- The absolute value lines are present to keep values positive

$$Area = \frac{1}{2}\left|x_1y_2 - x_2y_1\right|$$

Tables book page 18

Equation of a Line

Tables book page 18

$$y - y_1 = m(x - x_1)$$

This equation is used when you have a point and a slope and are asked for the equation
If you have 2 points you can use either in the equation

$$y = mx + c$$

This equation is used when you already have the equation and would like to get some information about the line.

1. m = the slope of the line
2. c = the y-intercept (where the line crosses the y axis)

The **X- intercept** can be found by making **y = 0** in either of the equations above and then finding x
The **Y-intercept** can be found by having an equation in the form y=mx + c or by making **x=0** and finding y

If you have 2 lines that are intersecting you can find the **point of intersection** (where the 2 lines meet) by using **simultaneous equations** (covered in algebra)

To show a **point is on a line**, substitute the X value of the point into the X value of the equation and do the same for y. If you get **0** (ie it satisfies the equation) then the point is on the line.

If you have the equation of a line in the form ax+by+c=0 then the slope will always be -a/b

Divisor of a Line

- We can be asked to divide a line in a given ratio.
- Lines can be divided internally, meaning the dividing point is in between the 2 points given or externally, meaning the dividing point is outside the given points
- Internal division can be done using a formula from your tables book or by translation
- External division, rarely required, can be done using translation

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

Tables book page 18

Angle between 2 Lines

- As the title suggests the formula below will help you find the angle that exists between 2 lines
- You get 2 solutions
- One is typically acute (<90 degrees)
- The other will be obtuse (>90 degrees)
- If you simply find one solution you can minus that solution from 180 to find the second angle
- The angle required depends on a graph or what's asked in the question

$$tan\theta = \frac{\pm m_1 - m_2}{1 + m_1 m_2}$$

Tables book page 19

- m1 is slope one
- m2 is slope two
- You have a plus and minus as their are 2 angles. The acute angle (<90) and the obtuse angle (>90)
- Tip: Put the plus and minus with the tan on the left and the equation is often easier to solve
- Calculator or tables book needed for tan calculation
- Remember from JC that Tan = Opposite/adjacent which is the same as rise/run, so tan is also the slope in degrees

Perpendicular Distance between a Line and a Point

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Tables book page 19

given a line ax + by + c=0 and the point (x1, y1)

- Here we see the | | modulus or absolute value lines.
- Remember that all answers are thus positive (They may have been positive or negative so this needs to be considered in answering questions where the distance is given)
- Very useful for questions involving circles and their tangents

Find the shortest distance between the following line and point:

The line T contains the points (-3,9) and (-7,4)
A. Find the equation of T
B. Investigate if the point (2,-6) lies on T

The line T contains the points (a,0) and (0,b)
A. Find the slope of T in terms of a and b
B. Find the equation of T in terms of a and b

2018 Paper 2 Q5

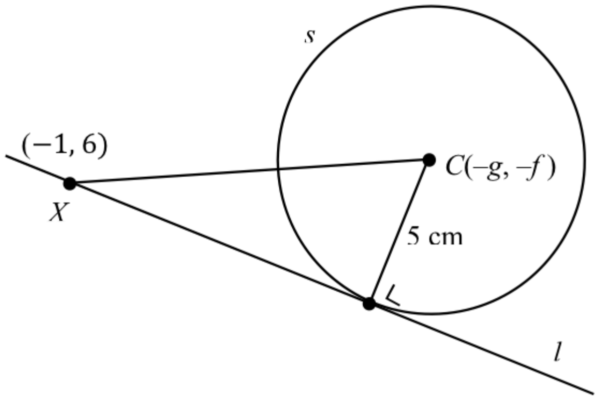
The line m: $2x + 3y + 1 = 0$ is parallel to the line n: $2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

2016 Paper 2 Q2

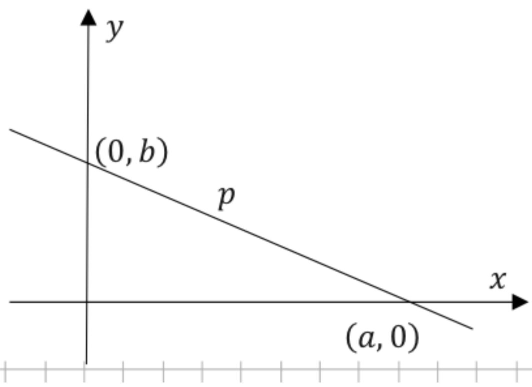
A point X has co-ordinates $(-1, 6)$ and the slope of the line XC is $\frac{1}{7}$.

(a) Find the equation of XC. Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.



2019 Paper 2 Q2

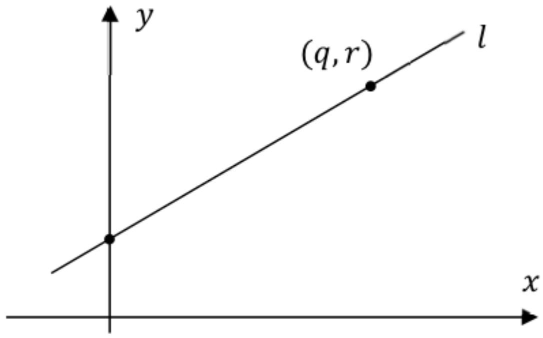
(a) The line p makes an intercept on the x-axis at $(a, 0)$ and on the y-axis at $(0, b)$, where $a, b \neq 0$.
Show that the equation of p can be written as $\frac{x}{a} + \frac{y}{b} = 1$.



(b) The line l has a slope m, and contains the point $A(6, 0)$.

(i) Write the equation of the line l in terms of m.

(b) The line l has a slope of m and contains the point (q, r), where $m, q, r \in \mathbb{R}$ are all positive. Find the co-ordinates of the point where l cuts the y-axis, in terms of m, q, and r.



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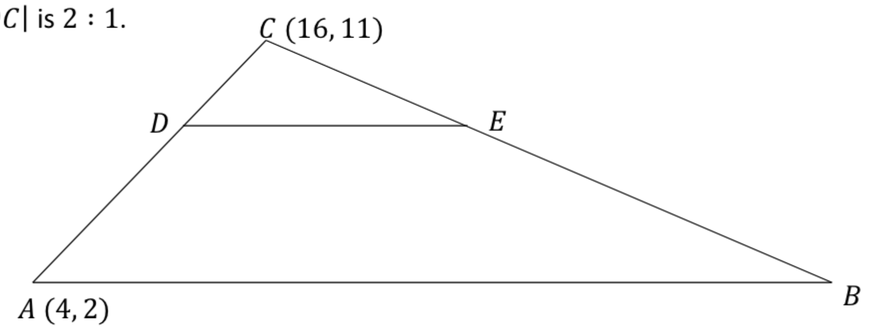
(c) The points $A(4, 2)$ and $C(16, 11)$ are vertices of the triangle ABC shown below.
D and E are points on $[CA]$ and $[CB]$ respectively.
The ratio $|AD| : |DC|$ is $2 : 1$.

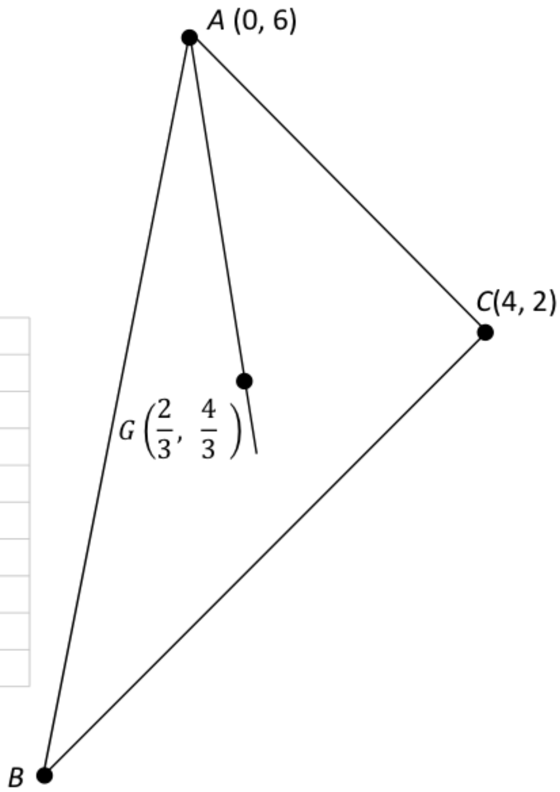
(i) Find $|AD|$.



(ii) $[AB]$and $[DE]$ are **horizontal** line segments.
$|AB| = 33$ units.
Find the coordinates of B and of E.

Centroid: Point of intersection of the lines connecting the midpoint of one line to the opposition vertex

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.
$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.
AG intersects *BC* at the point *P*.
$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

(b) Find the co-ordinates of *B*.

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- Then a second using equation 4 or 5
- Then the last variable using equation 1, 2 or 3
- Hint: if a variable is initially missing from one of your 3 equations then choose that variable to work with

$$a + b + c = 2 \quad (1)$$
$$2a + 3b + c = 7 \quad (2)$$
$$\frac{\cancel{6}^3 a}{\cancel{2}} - \frac{\cancel{6}b}{\cancel{6}} + \frac{\cancel{6}^2 c}{3} = \frac{\cancel{6}^2(2)}{\cancel{3}} = 3a - b + 2c = 4 \quad (3)$$

Solve the equation above:

c

(1) $\times -1$ $= -a - b - c = -2$
(2) $2a + 3b + c = 7$
$a + 2b = 5$ (4)

(1) $\times -2$ $-2a - 2b - 2c = -4$
(3) $3a - b + 2c = 4$
$a - 3b = 0$ (5)

(4) $\times -1$ $a + 2b = 5$
(5) $a - 3b = 0$

$-a - 2b = -5$
$a - 3b = 0$
$-5b = -5$
$\boxed{b = 1}$

$a - 3b = 0$
$a - 3(1) = 0$
$a - 3 = 0$
$\boxed{a = 3}$

$a + b + c = 2$
$3 + 1 + c = 2$
$4 + c = 2$
$\boxed{c = -2}$

2022 Paper 1 Q2

(ii) The areas of the three regions **K**, **L**, and **N** give the following three equations (including the equation from **part (b)(i)**):

$$
\begin{aligned}
4a + 3b + 3c &= 807 \\
28a + 9b + 3c &= 879 \\
76a + 15b + 3c &= 663
\end{aligned}
$$

Solve these equations to find the values of a, b, and c.

2018 Paper 1 Q 2

(a) Solve the simultaneous equations.

Variable choice of y

$$
\begin{aligned}
2x + 3y - z &= -4 \quad (1) \\
3x + 2y + 2z &= 14 \quad (2) \\
x \qquad - 3z &= -13 \quad (3)/(4)
\end{aligned}
$$

2013 Paper 1 Question 2

(b) Solve the simultaneous equations;

$$
\begin{aligned}
x + y + z &= 16 \\
\frac{5}{2}x + y + 10z &= 40 \\
2x + \frac{1}{2}y + 4z &= 21.
\end{aligned}
$$

Algebraically

- If we wish to solve for the point/s of intersection without graphing, then we use an algebraic approach

5 Steps to follow:

1. Look at the linear of the 2 equations (no square) and isolate one of the variables (Letters)
2. Substitute your now isolated expression into the non linear equation
3. Expand and simplify
4. Solve for your variable
5. Substitute in order to find the solutions for the second variable

Relevant again in the Circle when we look for a line intersecting a circle

$xy = 4$ non linear

$2x - y + 2 = 0$ linear

(1) $2x - y + 2 = 0$

$\boxed{2x + 2 = y}$ OR $\frac{2x}{2} = \frac{y-2}{2}$

$x = \frac{y-2}{2}$

(2) $xy = 4$

$x(2x+2) = 4$

(3) $2x^2 + 2x - 4 = 0$

$\div 2$

$x^2 + 1x - 2 = 0$

(4) $(x+2)(x-1) = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

(5) $y = 2x + 2$

$y = 2(-2) + 2$

$= -4 + 2 = \boxed{-2}$

$y = 2(1) + 2$

$= 2 + 2 = \boxed{4}$

$(-2, -2)$ $(1, 4)$

$x^2 + x - 2 = 0$

$a = 1$
$b = 1$
$c = -2$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$

(a) Solve the simultaneous equations:

$a^2 - ab + b^2 = 3$ non linear

$a + 2b + 1 = 0$ linear

① $a = -2b - 1$

② $(-2b-1)^2 - (-2b-1)b + b^2 = 3$

2016 paper 1 Q2

(b) Solve the simultaneous equations:

$x^2 + xy + 2y^2 = 4$ non-linear

$2x + 3y = -1.$ linear

① $\frac{2x}{2} = \frac{-1-3y}{2}$

$x = \frac{-1-3y}{2}$

② $\left(\frac{-1-3y}{2}\right)^2 + \left(\frac{-1-3y}{2}\right)y + 2y^2 = 4$

③ $4\frac{(1+6y+9y^2)}{4} + 4\frac{(-y-3y^2)}{2} + 4(2y^2) = (4)4$

$1 + 6y + 9y^2 + 2(-y-3y^2) + 8y^2 = 16$

Extra Q

$Pa^3 = 710$ $\quad$ $Pa^5 = 830$

① $\boxed{P = \frac{710}{a^3}}$

② $\left(\frac{710}{\cancel{a^3}}\right)\cancel{a^5}\,a^2 = 830$

$710a^2 = 830$

$a^3 = \frac{710}{P} = a = \sqrt[3]{\frac{710}{P}}$

Complex #5

$V = 2(\cos\frac{\pi}{6} + i\sin\pi/6$

$v^3 \; [2((\cos\pi/6 + i\sin\pi/6)]^3$

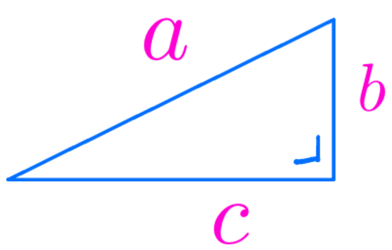
$2^3(\cos 3\pi/6 + i\sin 3\pi/6)$

Practical Trigonometry

Pythagoras:

Helps you find the side in a right angled triangle

You typically need to know 2 sides to find the third



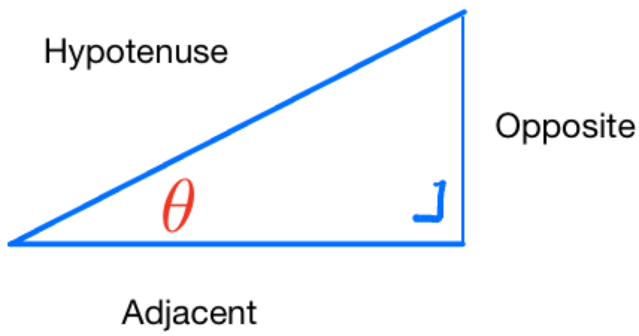
$$a^2 = b^2 + c^2$$

The Hypotenuse squared is equal to the sum of the squares of the remaining 2 sides

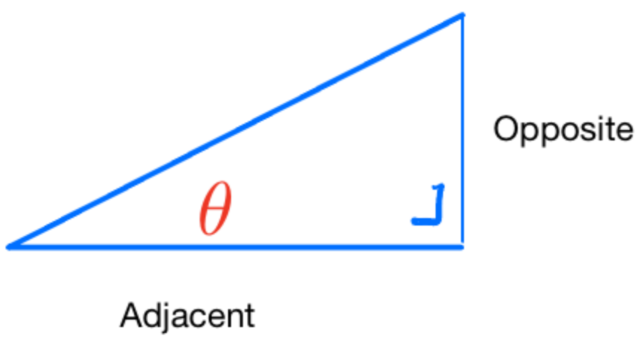
- It only works with right angled triangles
- The longest side is the one opposite the right angle and is known as the hypotenuse
- The remaining 2 sides have, up until now, been referred to as the height and base
- When we have 3 integers in the place of a, b and c we have a Pythagorean triple

Soh Cah Toa

θ = the Greek letter Theta. It is often used to represent an unknown angle

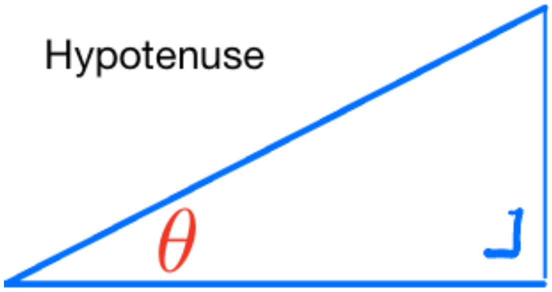


- Now lets start pairing up the sides
- There are 3 possible combinations



- When we have the opposite/adjacent we call this situation the Tangent (tan) of the angle
- We have 2 sides as a fraction thus creating a ratio

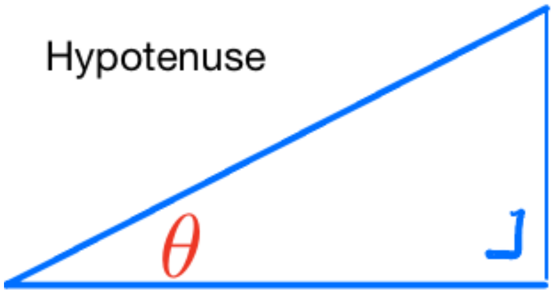
Hypotenuse



Adjacent

- When we have the adjacent/ hypotenuse we have the Cosine (Cos) of the angle

Hypotenuse



Opposite

- Lastly we have the Opposite/ hypotenuse which is called the Sine (Sin) of the angle

- These 3 ratios are the building blocks of trigonometry and cover all possible combinations in right angle triangles
- Once you have labelled the sides correctly you will know which ratio to use and then be able to convert your ratio into an angle

$\sin\theta$ = Opp/Hyp Soh

$\cos\theta$ = Adj/Hyp Cah

$\tan\theta$ = Opp/Adj Toa

<u>Soh Cah Toa</u>

Used to find missing sides or angles in right angled triangles

Area of a Triangle - Trigonometric formula

- Finding the area of a triangle is nothing new
- We have used half x base x height with right angled triangles
- We've also seen a formula in co-ordinate geometry of the line where we have 3 points and carry out a translation
- Now we look at having 2 sides of a non-right angled triangle (it will also work for right angled triangles) with the angle in the middle of those 2 sides
- For this to work we use the formula:

$$Area = \frac{1}{2}ab\sin C$$

Tables book page 16

- The small letters represent **sides**
- The capital letters represent **angles**
- Answers should be in square units

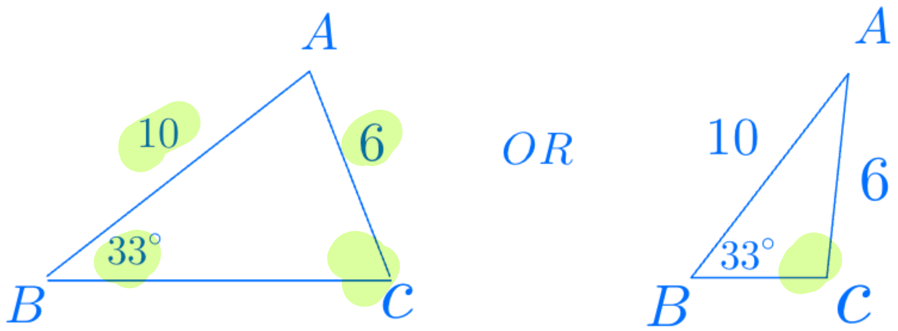
Sine Rule

- The Sine rule allows us to identify sides or angles in right angled and non right angled Triangles
- It allows us to do so by comparing opposite sides and angles
- You must know one side and it's opposite angle with one other side/angle for the sine rule to work
- If you do not have the necessary components then Cosine rule in the next section will typically be the answer
- Don't forget the area of a triangle above can help find missing sides and angles

$$\frac{a}{\sin A} = \frac{b}{sin B} = \frac{c}{\sin C}$$

Tables book page 16

- a, b and c refer to sides of a triangle
- A, B and C refer to the angles opposite the matching sides
- Can be used for right angled or non right angled triangles
- You need only look at 2 sides/angles at a time
- **When you are 2 sides and an angle not in between those sides you need to look for the ambiguous case.**



- Relevant when the diagram is not given
- You need to use the reference angles for sine to find the second possible angle

- Before we use Sine rule for problem solving I want to take a look at 2 formal proofs in Trigonometry
- A formal proof can be asked as a proof in the Leaving Certificate Exam

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

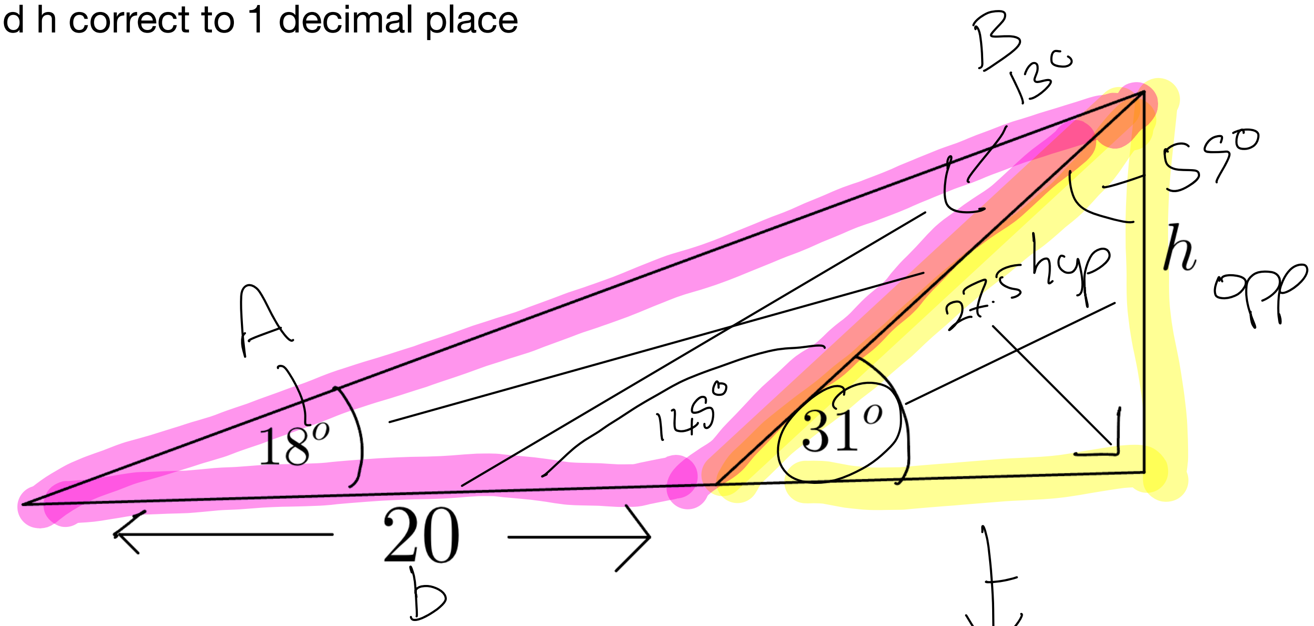
Tables book pg 15

- Again the small letters represent **sides**
- The capital letters represent **angles**
- Can be used in right angled and non right angled triangles
- Can be used when you have 3 sides and no angle

- The goal in practical trigonometry is to use any of the above rules when problem solving
- Try and look at the problem and then ask your yourself:

1. Are there any common sides?
2. Which triangle do I know the most about?
3. Are there any simple calculations I can do?
4. Which rule should I use to find the common side? ✓
5. How can I find any remaining sides or angles in the question?

Find h correct to 1 decimal place



$180 - 31 = 149°$

$180 - 149 - 18° = 13°$

$\frac{a}{\sin A} = \frac{b}{\cos B}$ $= \frac{a}{\cancel{\sin 18}} = \frac{20 \sin 18}{\sin 13}$

$180° - 90° - 31$

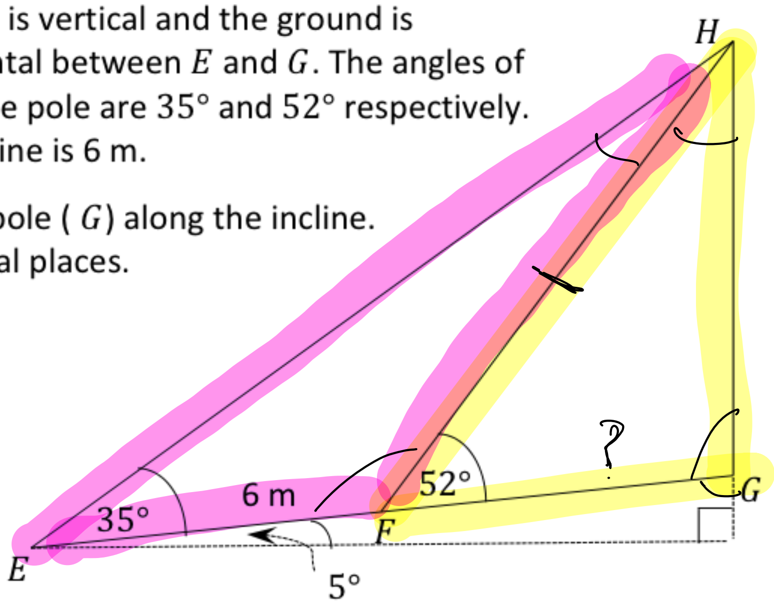
$a = \frac{20 \sin 18}{\sin 13} = 27.5$

Soh Cah Toa

$27.5 \sin 31 = \frac{h(27.5)}{27.5}$ $\therefore$ $h = 27.5 \sin 31$
$= 14.2$

(a) A flagpole $[GH]$, shown in the diagram, is vertical and the ground is inclined at an angle of $5°$ to the horizontal between E and G. The angles of elevation from E and F to the top of the pole are $35°$ and $52°$ respectively. The distance from E to F along the incline is 6 m.

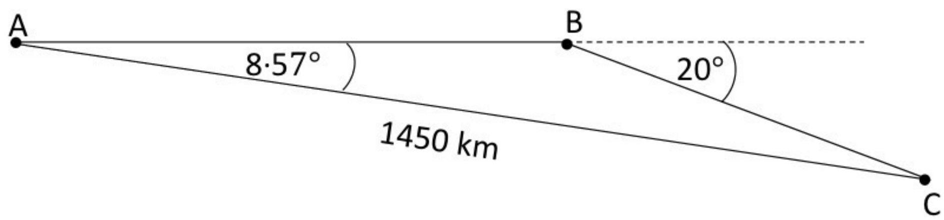
Find how far F is from the base of the pole (G) along the incline.
Give your answer correct to two decimal places.



2021 Paper 2 Q9

(a) An aeroplane flies east from point A for 2 hours at a constant speed of 420 km per hour until it reaches point B. It then changes direction by heading 20° towards the south at the same speed until it reaches point C, as shown in the diagram below.
The direct distance from A to C is 1450 km and $|\angle BAC| = 8{\cdot}57°$.

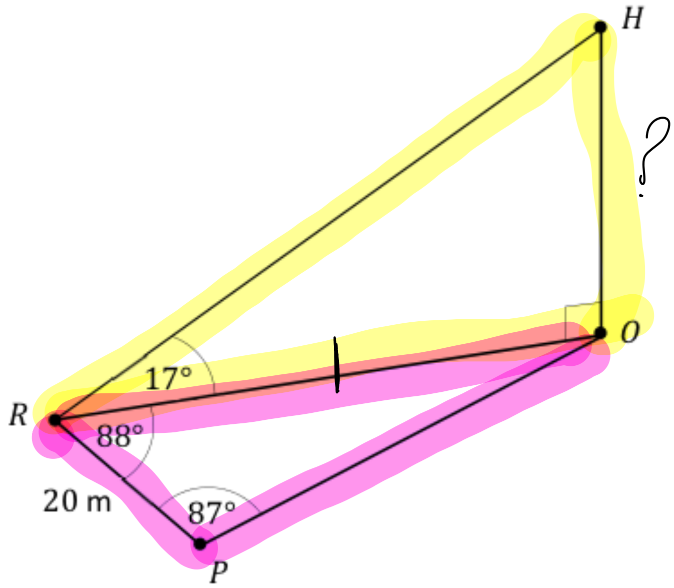
(i) Find how long it took to fly from B to C.
Give your answer correct to the nearest minute.



(b) Olga wants to measure the vertical height of a hill. The point H is at the top of the hill. The points R and P are 20 m apart on horizontal ground, at the bottom of the hill.

Olga measures the angle of elevation from R to H.
Taking O to be the point directly below H that is horizontal with R and P, Olga also measures the angles $\angle OPR$ and $\angle ORP$.
All of these are shown in the diagram below (not to scale).

Source: www.bikeforums.net/road-cycling



Work out the distance $|OH|$, the vertical height of the top of the hill relative to the points R and P. Give your answer correct to the nearest metre.

Co-ordinate Geometry of the Line

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Midpoint and Distance

- The midpoint is about finding the point in between (in the middle of) 2 other points
- This is done by finding the average of the 2 X values and the average of the 2 Y values
- Another phrase can be to find the point that Bisects a line
- Bisect means to cut in half, so the midpoint is involved

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Found on page 18 of the Tables Book

- For Distance or length of a line segment we can use the formula below
- Another approach can be to use The Theorem of Pythagoras

$$Length/Distace = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Found on page 18 of the Tables Book

Slope of a Line

- Another word for slope is gradient
- Here we can use our formula given in the tables book
- We can also look at the slope as rise over run
- This can be identified by making a right angled triangle out of our 2 points
- The height is the rise and the base is the run
- If we get a positive slope that means our line is increasing
- If we get a negative slope then our line is decreasing
- The symbol for slope is ‘m’

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} / \frac{rise}{run}$$

Found on page 18 of the Tables Book

$A \| B$ This is how you say 2 lines are parallel with symbols

$A \perp B$ This is the symbol we use to show 2 lines are perpendicular

- Lines can be parallel or perpendicular
- Parallel lines have the exact same slope
- Perpendicular lines are at 90 degrees to each other and their slopes multiply to always give -1
- m1 x m2 = -1 when 2 lines are perpendicular
- This can be used to find a perpendicular slope if you already know one slope
- Alternatively invert the slope you have and change it's sign

Translations and area of a Triangle

- We have used translations before in the functions handout, when used graphically
- Now we will use it with points
- This follows on nicely from finding the length and slope of a line
- This means moving an object, in this case one or more points
- Now we can use our ability to translate to work out the area of a Triangle
- This is a useful formula as it relies on using 3 points
- Other formulas will use lengths (1/2 x base x height) or lengths and angles (Trigonometry)
- One of our points must be (0,0) for this formula to work, therefore translation is often required
- Since we are finding area our answer must be positive and given in the correct units squared
- The absolute value lines are present to keep values positive

$$Area = \frac{1}{2}\left|x_1y_2 - x_2y_1\right|$$

Tables book page 18

Equation of a Line

Tables book page 18

$$y - y_1 = m(x - x_1)$$

This equation is used when you have a point and a slope and are asked for the equation
If you have 2 points you can use either in the equation

$$y = mx + c$$

This equation is used when you already have the equation and would like to get some information about the line.

1. m = the slope of the line
2. c = the y-intercept (where the line crosses the y axis)

The **X- intercept** can be found by making **y = 0** in either of the equations above and then finding x
The **Y-intercept** can be found by having an equation in the form y=mx + c or by making **x=0** and finding y

If you have 2 lines that are intersecting you can find the **point of intersection** (where the 2 lines meet) by using **simultaneous equations** (covered in algebra)

To show a **point is on a line**, substitute the X value of the point into the X value of the equation and do the same for y. If you get **0** (ie it satisfies the equation) then the point is on the line.

If you have the equation of a line in the form ax+by+c=0 then the slope will always be -a/b

Divisor of a Line

- We can be asked to divide a line in a given ratio.
- Lines can be divided internally, meaning the dividing point is in between the 2 points given or externally, meaning the dividing point is outside the given points
- Internal division can be done using a formula from your tables book or by translation
- External division, rarely required, can be done using translation

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

Tables book page 18

Angle between 2 Lines

- As the title suggests the formula below will help you find the angle that exists between 2 lines
- You get 2 solutions
- One is typically acute (<90 degrees)
- The other will be obtuse (>90 degrees)
- If you simply find one solution you can minus that solution from 180 to find the second angle
- The angle required depends on a graph or what's asked in the question

$$tan\theta = \frac{\pm m_1 - m_2}{1 + m_1 m_2}$$

Tables book page 19

- m1 is slope one
- m2 is slope two
- You have a plus and minus as their are 2 angles. The acute angle (<90) and the obtuse angle (>90)
- Tip: Put the plus and minus with the tan on the left and the equation is often easier to solve
- Calculator or tables book needed for tan calculation
- Remember from JC that Tan = Opposite/adjacent which is the same as rise/run, so tan is also the slope in degrees

Perpendicular Distance between a Line and a Point

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Tables book page 19

given a line ax + by + c=0 and the point (x1, y1)

- Here we see the | | modulus or absolute value lines.
- Remember that all answers are thus positive (They may have been positive or negative so this needs to be considered in answering questions where the distance is given)
- Very useful for questions involving circles and their tangents

Find the shortest distance between the following line and point:

The line T contains the points $\overset{x_1, y_1}{(-3,9)}$ and $\overset{x_2\ y_2}{(-7,4)}$

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A. Find the equation of T

B. Investigate if the point (2,-6) lies on T

A) $y - y_1 = m(x - x_1)$

Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{4-9}{-7-(-3)} = \dfrac{5}{4}$

$4(y-9) = \dfrac{5}{\cancel{4}}(x-(-3))$

$4y - 36 = 5x + 15$

$0 = 5x - 4y + 51$

B) $0 = 5(2) - 4(-6) + 51$

$10 + 24 + 51$

$0 \neq 85 \quad \therefore (2,-6)$ does not lie on T

The line T contains the points $\overset{x_1, y_1}{(a,0)}$ and $\overset{x_2\ y_2}{(0,b)}$

A. Find the slope of T in terms of a and b

B. Find the equation of T in terms of a and b

A) $\dfrac{y_2 - y_1}{x_2 - x_1} =$ slope $= m$

$\dfrac{b-0}{0-a} = \dfrac{b}{-a} = \dfrac{-b}{a}$

B) $y - y_1 = m(x - x_1)$

$a(y - 0) = \cancel{a}\left(\dfrac{-b}{\cancel{a}}\right)(x - a)$

$ay = -bx + ab$

$bx + ay - ab = 0$

$ax + by + c = 0$

2018 Paper 2 Q5

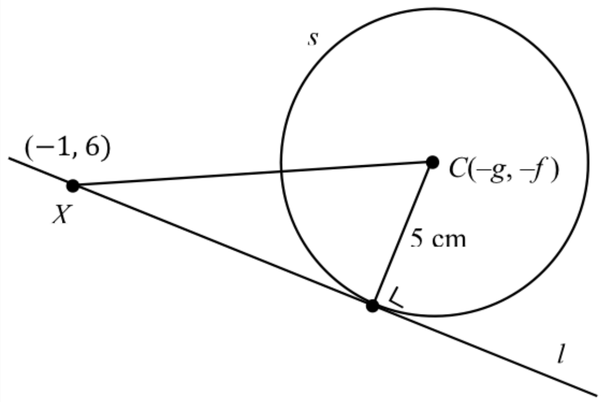
The line $m: 2x + 3y + 1 = 0$ is parallel to the line $n: 2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

2016 Paper 2 Q2

A point X has co-ordinates $(-1, 6)$ and the slope of the line XC is $\frac{1}{7}$.

(a) Find the equation of XC. Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.



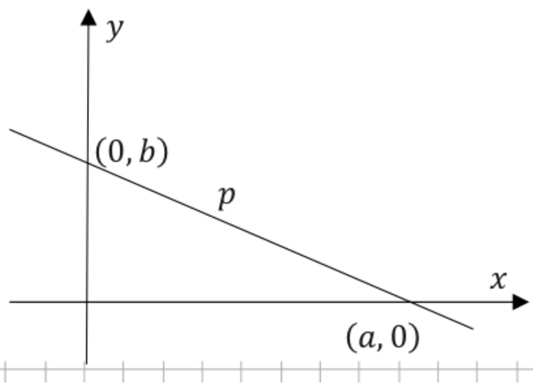
2019 Paper 2 Q2

(a) The line p makes an intercept on the x-axis at $(a, 0)$ and on the y-axis at $(0, b)$, where $a, b \neq 0$.

Show that the equation of p can be written as $\frac{x}{a} + \frac{y}{b} = 1$.

$$\frac{\cancel{b}x}{a\cancel{b}} + \frac{\cancel{a}y}{\cancel{a}b} = \frac{\cancel{ab}}{\cancel{ab}}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

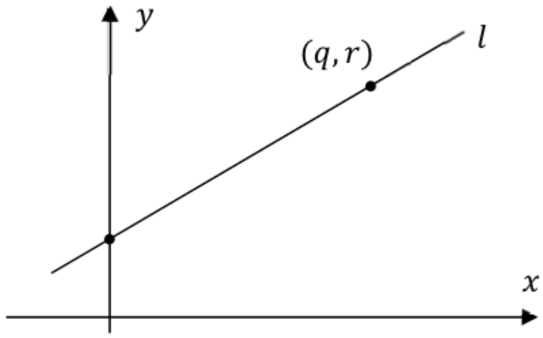


(b) The line l has a slope m, and contains the point $A(6, 0)$.

(i) Write the equation of the line l in terms of m.

(b) The line l has a slope of m and contains the point (q, r), where $m, q, r \in \mathbb{R}$ are all positive.
Find the co-ordinates of the point where l cuts the y-axis, in terms of m, q, and r.

$x = 0$ $\quad y - y_1 = m(x - x_1)$

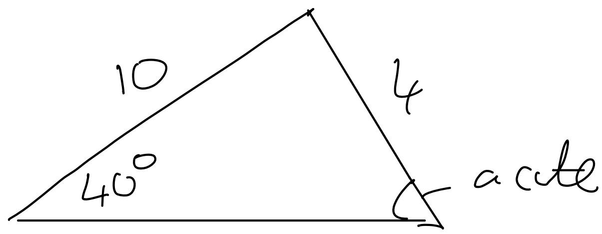


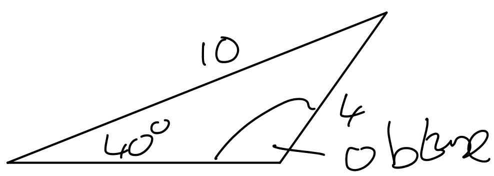
P1

Algebra
Logs
Indices
Functions
Calculus
Seqs + Series
Complex #
Financial Maths
Induction

P2

Trigonometry
Statistics
Probability
Co-ordinate geometry
Synthetic geometry

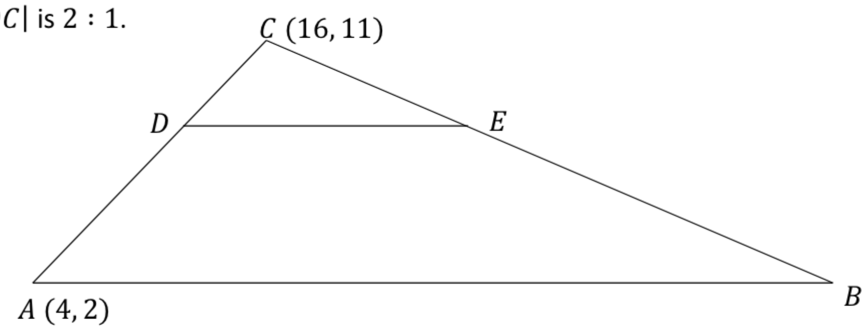




2021 Paper 2 Q2

(c) The points $A(4, 2)$ and $C(16, 11)$ are vertices of the triangle ABC shown below.
D and E are points on $[CA]$ and $[CB]$ respectively.
The ratio $|AD| : |DC|$ is $2 : 1$.

(i) Find $|AD|$.



(ii) $[AB]$and $[DE]$ are **horizontal** line segments.
$|AB| = 33$ units.
Find the coordinates of B and of E.

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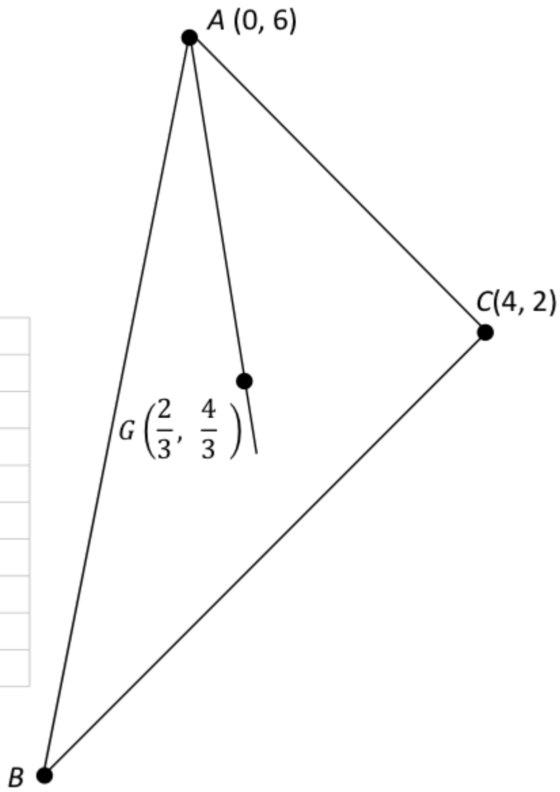
Centroid: Point of intersection of the lines connecting the midpoint of one line to the opposition vertex

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.
$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.
AG intersects *BC* at the point *P*.
$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

(b) Find the co-ordinates of *B*.

$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

① Show true for $n = 1$

$1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1$ ✓

② Assume true for $n = k$

$1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$

$\frac{5}{10}$

③ Prove true for $n = k+1$

$1 + 2 + 3 + \dots + k + k + 1 = \frac{(k+1)(k+1+1)}{2} = \frac{(k+1)(k+2)}{2}$

$\frac{k(k+1)}{2} + k + 1$

$\frac{k(k+1)}{2} + \frac{2(k+1)}{2}$

$\frac{k(k+1) + 2(k+1)}{2}$

$\frac{(k+1)(k+2)}{2}$

④ True $n = 1$, Assumed true for $n = k$

Hence $P(n)$ true for all values of n

2023 P1 SA Q2.B

$(\log_2 3)(\log_3 4)(\log_4 5)\ldots(\log_{n-1} n)(\log_n (n+1)) = 11$

$\log_2 3 \ldots\ldots \frac{\log_2 n+1}{\log_2 n} = 11$

$\log_2 n+1 = 11$

$2^{11} = n+1$

$2^{11} - 1 = n$

$2047 = n$