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Algebra Simultaneous Equations

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Solving 3 X Simultaneous Equations

- Remove fractions if present
- Number all 3 equations 1, 2 and 3 ✓
- Pick 2 of the 3 equations
- Choose 1 variable
- In both equations ensure that the chosen variable has the same coefficient with opposite signs in each equation
- Add the 2 equations
- The resulting equation is number 4
- Repeat from the beginning, choosing 2 different equations but the same variable
- Now you have equation 5
- Treat equations 4 and 5 as in previous studies
- Find one variable
- Then a second using equation 4 or 5
- Then the last variable using equation 1, 2 or 3
- Hint: if a variable is initially missing from one of your 3 equations then choose that variable to work with

$a + b + c = 2$ (1)

$2a + 3b + c = 7$ (2)

$\frac{a}{2} - \frac{b}{6} + \frac{c}{3} = \frac{2}{3}$ (3) $= 3a - b + 2c = 4$

$\boxed{b}$

Solve the equation above:

(1) $a + b + c = 2$

(3) $3a - b + 2c = 4$

$4a + 3c = 6$ (4)

(2) $2a + 3b + c = 7$

(3) $3a - b + 2c = 4 \quad | \times 3$

$2a + 3b + c = 7$

$9a - 3b + 6c = 12$

$11a + 7c = 19$ (5)

$4a + 3c = 6 \quad \times -7$

$11a + 7c = 19 \quad \times 3$

$-28a - \cancel{21c} = -42$

$33a + \cancel{21c} = +57$

$5a = 15 \quad | \div 5$

$\boxed{a = 3}$

$4(3) + 3c = 6$

$12 + 3c = 6$

$3c = -6$

$\boxed{c = -2}$

$3 + b - 2 = 2$

$\boxed{b = 1}$

2022 Paper 1 Q2

(ii) The areas of the three regions **K, L,** and **N** give the following three equations (including the equation from **part (b)(i)**):

$$4a + 3b + 3c = 807 \quad (1)$$
$$28a + 9b + 3c = 879 \quad (2)$$
$$76a + 15b + 3c = 663 \quad (3)$$

Isolate c

Solve these equations to find the values of a, b, and c.

2013 Paper 1 Question 2

(b) Solve the simultaneous equations;

$$x + y + z = 16$$
$$2\left(\frac{5}{2}\right)x + 2(y) + 2(10z) = 2(40) = 5x + 2y + 20z = 80$$
$$2(2x) + 4\left(\frac{1}{2}\right)y + 2(4z) = 2(21) \Rightarrow 4x + y + 8z = 42$$

2018 Paper 1 Q 2

$\boxed{5}$

(a) Solve the simultaneous equations.

$$2x + 3y - z = -4 \quad (1)$$
$$3x + 2y + 2z = 14 \quad (2)$$
$$x - 3z = -13 \quad (3) = (4)$$

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Algebraically

Function
Circle
Integration

- If we wish to solve for the point/s of intersection without graphing, then we use an algebraic approach

5 Steps to follow:
1. Look at the linear of the 2 equations (no square) and isolate one of the variables (Letters) ✓
2. Substitute your now isolated expression into the non linear equation ✓
3. Expand and simplify ✓
4. Solve for your variable ✓
5. Substitute in order to find the solutions for the second variable

Relevant again in the Circle when we look for a line intersecting a circle

$$xy = 4$$

$$2x - y + 2 = 0$$

$ax + by + c = 0$

(1) $2x - y + 2 = 0$

$2x + 2 = y$ ✓

(2) $xy = 4$

$x(2x + 2) = 4$

(3) $2x^2 + 2x - 4 = 0$

(4) $-b$

$2x^2 + 2x - 4 = 0$

$\div 2 \longrightarrow$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2 \quad x = 1$

(5) $y = 2x + 2$

$y = 2(-2) + 2$

$y = -4 + 2$

$y = -2$

$y = 2(1) + 2$

$y = 2 + 2$

$y = 4$

Points of Intersection

$(-2, -2)$ $(1, 4)$

2012 Paper 1 Q1

(a) Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$
$$a + 2b + 1 = 0$$

(1) = $a = -2b - 1$

2016 paper 1 Q2

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

(1)

$2x = -3y - 1$

$x = \frac{-3y-1}{2}$

(2) $\left(\frac{-3y-1}{2}\right)^2 + \left(\frac{-3y-1}{2}\right)y + 2y^2 = 4$

(3) $^4\frac{(9y^2+6y+1)}{4} + {}^4\frac{(-3y^2-y)}{2} + {}^4(2y^2) = (4)^4$

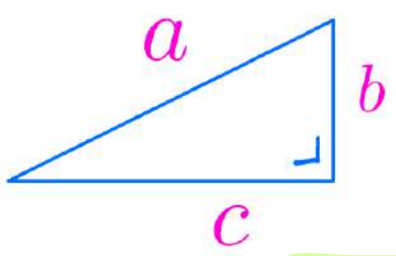
Practical Trigonometry

Pythagoras:

Helps you find the side in a right angled triangle

You typically need to know 2 sides to find the third

Complex numbers



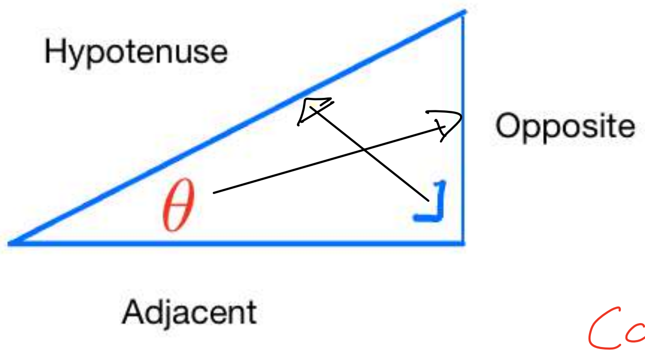
$$a^2 = b^2 + c^2$$

The Hypotenuse squared is equal to the sum of the squares of the remaining 2 sides

- It only works with right angled triangles
- The longest side is the one opposite the right angle and is known as the hypotenuse
- The remaining 2 sides have, up until now, been referred to as the height and base
- When we have 3 integers in the place of a, b and c we have a Pythagorean triple

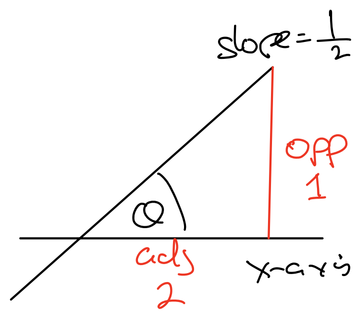
Soh Cah Toa

θ = the Greek letter Theta. It is often used to represent an unknown angle

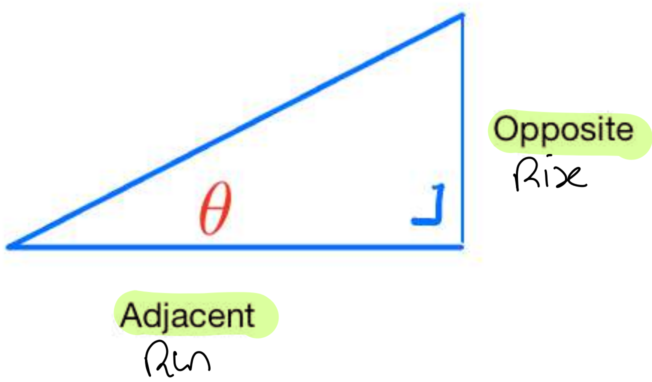


Calculus

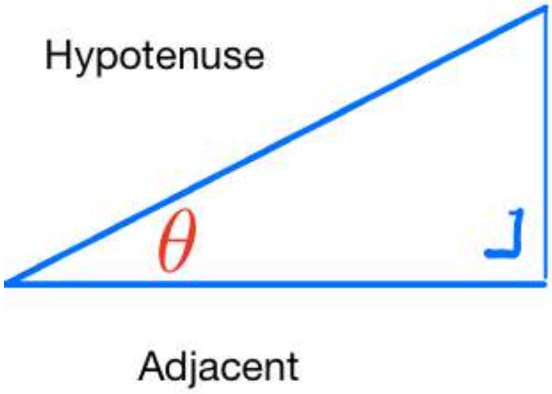
$$\text{Slope} = \frac{\text{Rise}}{\text{Run}}$$
$$\tan\theta = \frac{\text{Opp}}{\text{adj}}$$



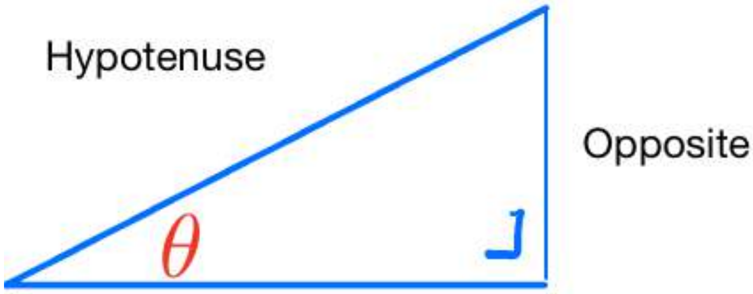
- Now lets start pairing up the sides
- There are 3 possible combinations



- When we have the opposite/adjacent we call this situation the Tangent (tan) of the angle
- We have 2 sides as a fraction thus creating a ratio



- When we have the adjacent/ hypotenuse we have the Cosine (Cos) of the angle



- Lastly we have the Opposite/ hypotenuse which is called the Sine (Sin) of the angle

- These 3 ratios are the building blocks of trigonometry and cover all possible combinations in right angle triangles
- Once you have labelled the sides correctly you will know which ratio to use and then be able to convert your ratio into an angle

$\sin\theta$ =	Opp/Hyp	Soh
$\cos\theta$ =	Adj/Hyp	Cah
$\tan\theta$ =	Opp/Adj	Toa

Soh Cah Toa

Used to find missing sides or angles in right angled triangles

Area of a Triangle - Trigonometric formula

- Finding the area of a triangle is nothing new
- We have used half x base x height with right angled triangles
- We've also seen a formula in co-ordinate geometry of the line where we have 3 points and carry out a translation
- Now we look at having 2 sides of a non-right angled triangle (it will also work for right angled triangles) with the angle in the middle of those 2 sides
- For this to work we use the formula:

$$Area = \frac{1}{2}ab\sin C$$

Tables book page 16

- The small letters represent **sides**
- The capital letters represent **angles**
- Answers should be in square units

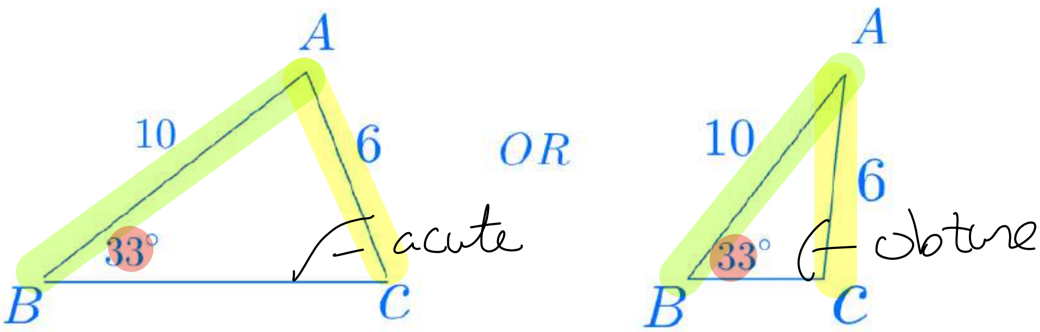
Sine Rule

- The Sine rule allows us to identify sides or angles in right angled and non right angled Triangles
- It allows us to do so by comparing opposite sides and angles
- You must know one side and it's opposite angle with one other side/angle for the sine rule to work
- If you do not have the necessary components then Cosine rule in the next section will typically be the answer
- Don't forget the area of a triangle above can help find missing sides and angles

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \cancel{\frac{c}{\sin C}}$$

Tables book page 16

- a, b and c refer to sides of a triangle
- A, B and C refer to the angles opposite the matching sides
- Can be used for right angled or non right angled triangles
- You need only look at 2 sides/angles at a time
- **<u>When you are 2 sides and an angle not in between those sides you need to look for the ambiguous case.</u>**



- Relevant when the diagram is not given
- You need to use the reference angles for sine to find the second possible angle

- Before we use Sine rule for problem solving I want to take a look at 2 formal proofs in Trigonometry
- A formal proof can be asked as a proof in the Leaving Certificate Exam

<u>Cosine Rule</u>

$$a^2 = b^2 + c^2 - 2bc\cos A$$

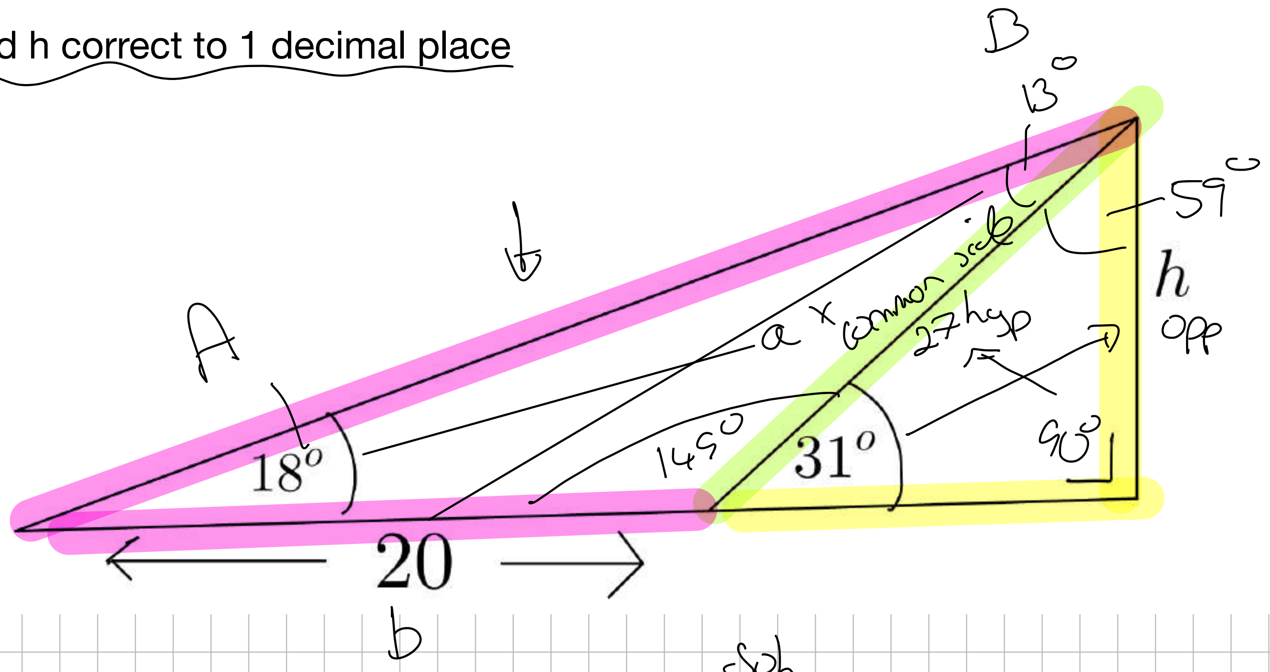
Tables book pg 15

- Again the small letters represent **sides**
- The capital letters represent **angles**
- Can be used in right angled and non right angled triangles
- Can be used when you have 3 sides and no angle

- The goal in practical trigonometry is to use any of the above rules when problem solving
- Try and look at the problem and then ask your yourself:

1. Are there any common sides?
2. Which triangle do I know the most about? ✓
3. Are there any simple calculations I can do? ✓
4. Which rule should I use to find the common side? ✓
5. How can I find any remaining sides or angles in the question?

Find h correct to 1 decimal place



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x\cancel{\sin 18}}{\cancel{\sin 18}} = \frac{20\sin 18}{\sin 13}$$

$$x = 27$$

SOH

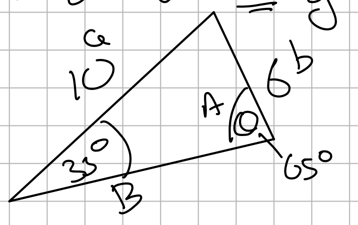
$$27\sin 31 = \frac{h}{\cancel{27}}\,(27)$$

$$14 = h$$

$$\boxed{\frac{48(2x)}{3} + \frac{48(x)}{2} = \frac{48(1)}{8}}$$

(3)(2)(8)
48

Find the values of θ



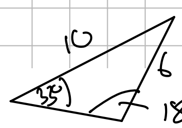
$$\frac{10}{\sin\theta} = \frac{6}{\sin 33}$$

$$\frac{10\sin\theta}{10} = \frac{10\sin 33}{6}$$

$$\sin\theta = \frac{10\sin 33}{6}$$

$$\theta = \sin^{-1}\left(\frac{10\sin 33}{6}\right)$$

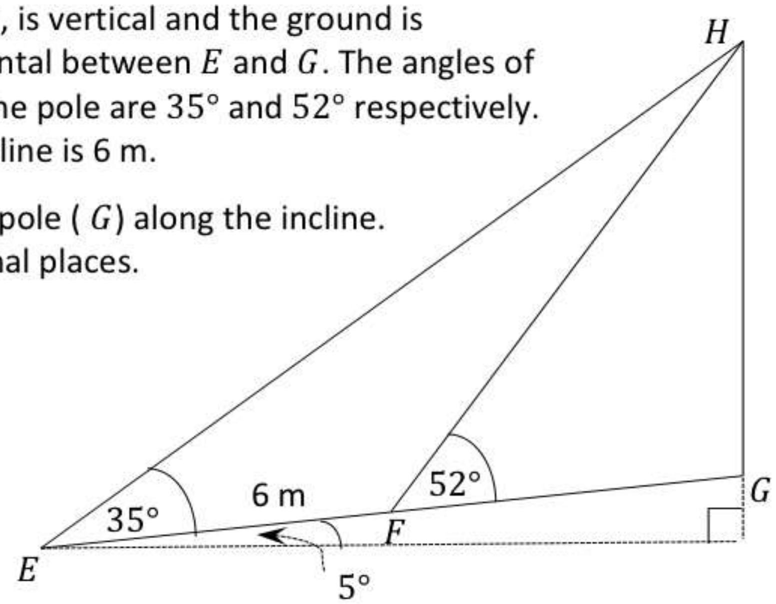
$$\theta = 65°$$



180° − 65°
115°

(a) A flagpole $[GH]$, shown in the diagram, is vertical and the ground is inclined at an angle of $5°$ to the horizontal between E and G. The angles of elevation from E and F to the top of the pole are $35°$ and $52°$ respectively. The distance from E to F along the incline is 6 m.

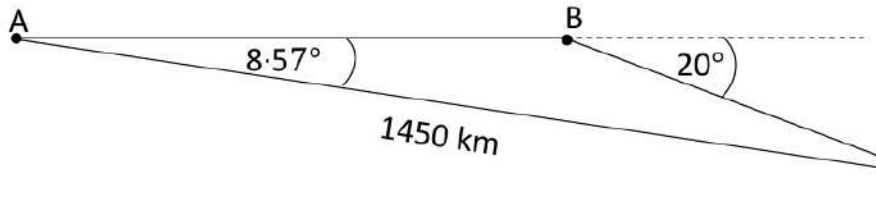
Find how far F is from the base of the pole (G) along the incline. Give your answer correct to two decimal places.



2021 Paper 2 Q9

(a) An aeroplane flies east from point A for 2 hours at a constant speed of 420 km per hour until it reaches point B. It then changes direction by heading 20° towards the south at the same speed until it reaches point C, as shown in the diagram below.
The direct distance from A to C is 1450 km and $|\angle BAC| = 8{\cdot}57°$.

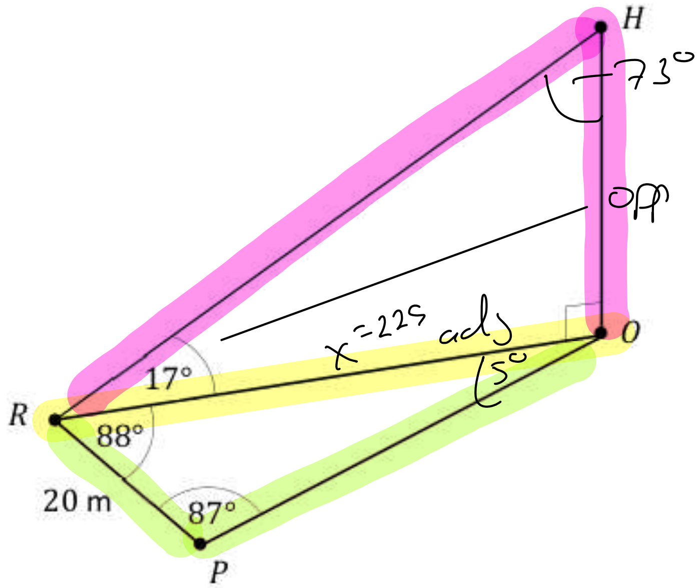
(i) Find how long it took to fly from B to C.
Give your answer correct to the nearest minute.



(b) Olga wants to measure the vertical height of a hill. The point H is at the top of the hill. The points R and P are 20 m apart on horizontal ground, at the bottom of the hill.

Olga measures the angle of elevation from R to H.
Taking O to be the point directly below H that is horizontal with R and P, Olga also measures the angles $\angle OPR$ and $\angle ORP$.
All of these are shown in the diagram below (not to scale).

Source: www.bikeforums.net/road-cycling



Work out the distance $|OH|$, the vertical height of the top of the hill relative to the points R and P. Give your answer correct to the nearest metre.

Co-ordinate Geometry of the Line

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Midpoint and Distance

- The midpoint is about finding the point in between (in the middle of) 2 other points
- This is done by finding the average of the 2 X values and the average of the 2 Y values
- Another phrase can be to find the point that Bisects a line
- Bisect means to cut in half, so the midpoint is involved

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Found on page 18 of the Tables Book

- For Distance or length of a line segment we can use the formula below
- Another approach can be to use The Theorem of Pythagoras

$$Length/Distace = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Found on page 18 of the Tables Book

Slope of a Line

- Another word for slope is gradient
- Here we can use our formula given in the tables book
- We can also look at the slope as rise over run
- This can be identified by making a right angled triangle out of our 2 points
- The height is the rise and the base is the run
- If we get a positive slope that means our line is increasing
- If we get a negative slope then our line is decreasing
- The symbol for slope is 'm'

– Calculus

$$Slope = \frac{y_2 - y_1}{x_2 - x_1} / \frac{rise}{run}$$

Found on page 18 of the Tables Book

$A||B$ This is how you say 2 lines are parallel with symbols

$A \perp B$ This is the symbol we use to show 2 lines are perpendicular

- Lines can be parallel or perpendicular
- Parallel lines have the exact same slope
- Perpendicular lines are at 90 degrees to each other and their slopes multiply to always give -1 — orthocentre
- m1 x m2 = -1 when 2 lines are perpendicular
- This can be used to find a perpendicular slope if you already know one slope
- Alternatively invert the slope you have and change it's sign

Translations and area of a Triangle

- We have used translations before in the functions handout, when used graphically
- Now we will use it with points
- This follows on nicely from finding the length and slope of a line
- This means moving an object, in this case one or more points
- Now we can use our ability to translate to work out the area of a Triangle
- This is a useful formula as it relies on using 3 points
- Other formulas will use lengths (1/2 x base x height) or lengths and angles (Trigonometry)
- One of our points must be (0,0) for this formula to work, therefore ~~translation~~ function is often required
- Since we are finding area our answer must be positive and given in the correct units squared
- The absolute value lines are present to keep values positive

$$Area = \frac{1}{2}|x_1y_2 - x_2y_1|$$ modulus

Tables book page 18

Equation of a Line

Tables book page 18

$$y - y_1 = m(x - x_1)$$

This tangent equation is used when you have a point and a slope and are asked for the equation
If you have 2 points you can use either in the equation

$$y = mx + c$$

This equation is used when you already have the equation and would like to get some information about the line.

1. m = the slope of the line
2. c = the y-intercept (where the line crosses the y axis) (0, c)

The **X- intercept** can be found by making **y = 0** in either of the equations above and then finding x
The **Y-intercept** can be found by having an equation in the form y=mx + c or by making **x=0** and finding y

If you have 2 lines that are intersecting you can find the **point of intersection** (where the 2 lines meet) by using **simultaneous equations** (covered in algebra)

To show a **point is on a line**, substitute the X value of the point into the X value of the equation and do the same for y. If you get **0** (ie it satisfies the equation) then the point is on the line.

If you have the equation of a line in the form ax+by+c=0 then the slope will always be -a/b

Divisor of a Line

- We can be asked to divide a line in a given ratio.
- Lines can be divided internally, meaning the dividing point is in between the 2 points given or externally, meaning the dividing point is outside the given points
- Internal division can be done using a formula from your tables book or by translation
- External division, rarely required, can be done using translation

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

Tables book page 18

Angle between 2 Lines

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- As the title suggests the formula below will help you find the angle that exists between 2 lines
- You get 2 solutions
- One is typically acute (<90 degrees)
- The other will be obtuse (>90 degrees)
- If you simply find one solution you can minus that solution from 180 to find the second angle
- The angle required depends on a graph or what's asked in the question

$$tan\theta = \frac{\pm m_1 - m_2}{1 + m_1 m_2}$$

Tables book page 19

- m1 is slope one
- m2 is slope two
- You have a plus and minus as their are 2 angles. The acute angle (<90) and the obtuse angle (>90)
- Tip: Put the plus and minus with the tan on the left and the equation is often easier to solve
- Calculator or tables book needed for tan calculation
- Remember from JC that Tan = Opposite/adjacent which is the same as rise/run, so tan is also the slope in degrees

Perpendicular Distance between a Line and a Point

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Tables book page 19

Radius - The circle

| | => modulus

given a line ax + by + c=0 and the point (x1, y1)

- Here we see the | | modulus or absolute value lines.
- Remember that all answers are thus positive (They may have been positive or negative so this needs to be considered in answering questions where the distance is given)
- Very useful for questions involving circles and their tangents

Find the shortest distance between the following line and point:

x_1, y_1 $\quad$ $x_2\ y_2$

The line T contains the points (-3,9) and (-7,4)

A. Find the equation of T ✓

B. Investigate if the point (2,-6) lies on T

$y - y_1 = m(x - x_1)$

$\dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{4-9}{-7-(-3)} = \dfrac{-5}{-4} = \dfrac{5}{4}$

$4\,(y - 9) = \dfrac{5}{\cancel{4}}(x - (-3))$

$4y - 36 = 5x + 15$

$0 = 5x - 4y + 51$

B. $5(2) - 4(-6)$

$10 + 24 + 51 \neq 0$

$\therefore$ (-2,-6) is not on the line T

$x\ y_1$ $\quad$ $x_2\ y_2$

The line T contains the points (a,0) and (0,b)

A. Find the slope of T in terms of a and b

B. Find the equation of T in terms of a and b

$y - y_1 = m(x - x_1)$

$\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{b-0}{0-a} = \dfrac{b}{-a} = \dfrac{-b}{a}$

$y - 0 = \dfrac{-b}{a}(x - a)$

$y = -\dfrac{bx}{a} + \dfrac{b\cancel{a}}{\cancel{a}}$

$y = -\dfrac{b}{a}x + b$

2018 Paper 2 Q5

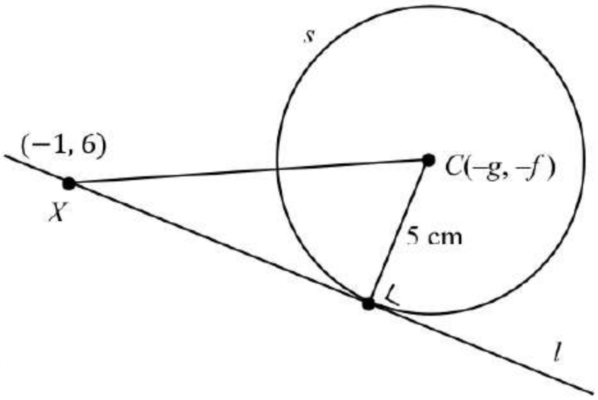
The line $m: 2x + 3y + 1 = 0$ is parallel to the line $n: 2x + 3y - 51 = 0$.

(a) Verify that $A(-2, 1)$ is on m.

2016 Paper 2 Q2

A point X has co-ordinates $(-1, 6)$ and the slope of the line XC is $\frac{1}{7}$.

(a) Find the equation of XC. Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.



2019 Paper 2 Q2

(a) The line p makes an intercept on the x-axis at $(a, 0)$ and on the y-axis at $(0, b)$, where $a, b \neq 0$.

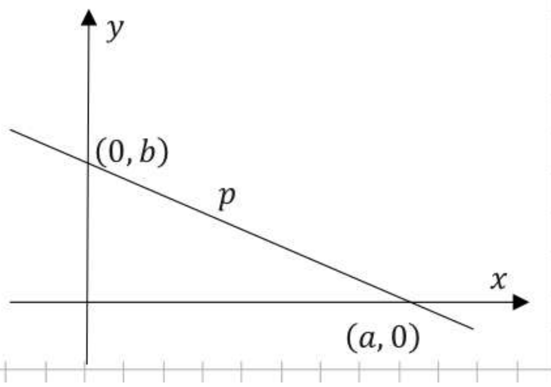
Show that the equation of p can be written as $\frac{x}{a} + \frac{y}{b} = \boxed{1.}$

$a(y) = \frac{a(-bx)}{a} + a1(b)$

$ay = -bx + ab$

$\frac{bx}{ab} + \frac{ay}{ab} = \frac{ab}{ab}$

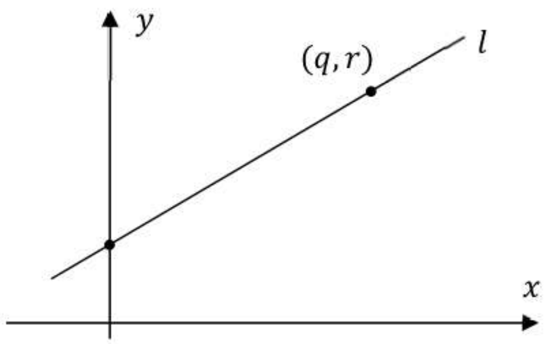
$\frac{x}{a} + \frac{y}{b} = 1$ ✓

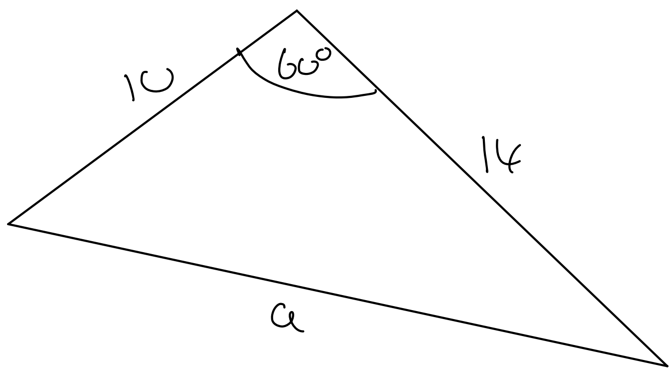


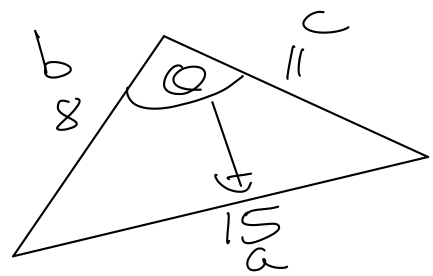
(b) The line l has a slope m, and contains the point $A(6, 0)$.

(i) Write the equation of the line l in terms of m.

(b) The line l has a slope of m and contains the point (q, r), where $m, q, r \in \mathbb{R}$ are all positive. Find the co-ordinates of the point where l cuts the y-axis, in terms of m, q, and r.







$a^2 = b^2 + c^2 - 2bc\cos A$

$\sqrt{a^2} = \sqrt{10^2 + (14)^2 - 2(10)(14)\cos 60}$

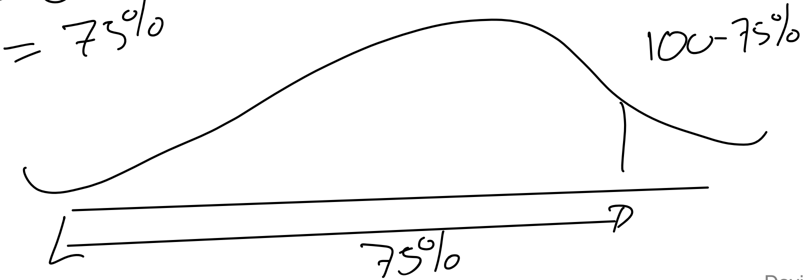
$a^2 = b^2 + c^2 - 2bc\cos A$

$\dfrac{a^2 - b^2 - c^2}{-2bc} = \dfrac{-2bc\cos A}{-2bc}$

$\dfrac{a^2 - b^2 - c^2}{-2bc} = \cos A$

$Z = 0.73$

$P = 75\%$



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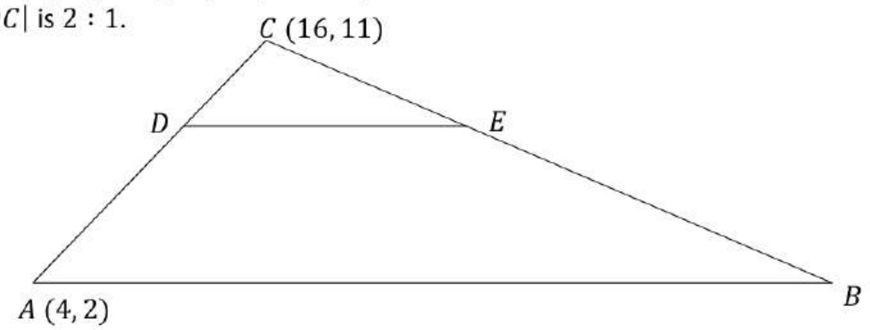
(c) The points $A(4, 2)$ and $C(16, 11)$ are vertices of the triangle ABC shown below.
D and E are points on $[CA]$ and $[CB]$ respectively.
The ratio $|AD| : |DC|$ is $2 : 1$.

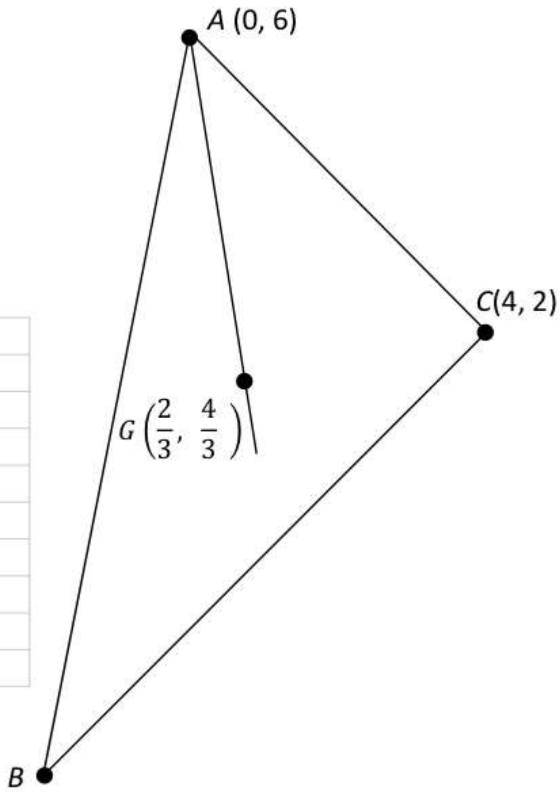
(i) Find $|AD|$.



(ii) $[AB]$and $[DE]$ are **horizontal** line segments.
$|AB| = 33$ units.
Find the coordinates of B and of E.

Centroid: Point of intersection of the lines connecting the midpoint of one line to the opposition vertex

ABC is a triangle where the co-ordinates of A and C are $(0, 6)$ and $(4, 2)$ respectively.
$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle ABC.
AG intersects BC at the point P.
$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of P.

(b) Find the co-ordinates of B.

Paper 1
Algebra
Induction
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Calculus (A+D)
Integration

Paper 2
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Statistics

$$\left(\sqrt{x+2}\right)^2 = \left(\sqrt{x^2-1}\right)^2$$

$$x+2 = x^2-1$$

High tide = 5.5m
Low tide = 1.7m

$[1.7, 5.5]$

Amplitude
1.9

$1.7 \leftarrow 3.6 \rightarrow 5.5$

$a + b\cos\theta$

$a + \underbrace{1.9\cos\theta}$

$[-1.9, 1.9]$
$+3.6$

$3.6 + 1.9\cos\theta$