THE INSTITUTE OF EDUCATION

Subject: Mathematics

TEACHER: Brian Scully

COURSE: Easter Revision

ACADEMIC LEVEL: Higher

ACADEMIC YEAR: $2025 - 2026$

TOPIC: Trigonometry – Practical Trigonometry

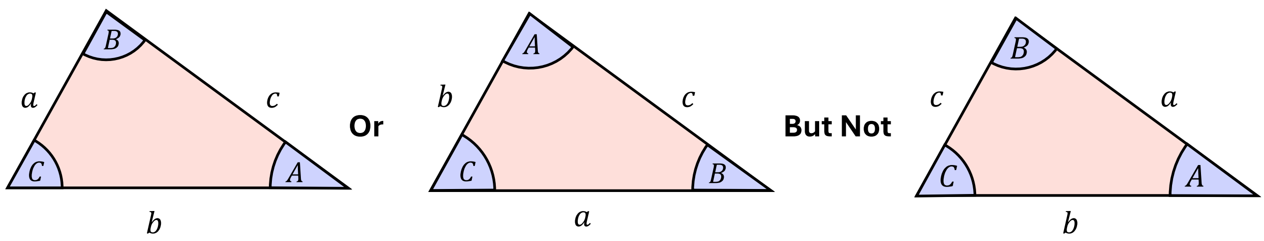
Unauthorised publication, distribution or reproduction of these notes is prohibited.

Solving Non-right Angled Triangles

Up until now, we have only dealt with triangles with contained a 90° angle. It is essential to recognise the formulae we used are only applicable to triangles which contain a right angle. For triangles which do not contain a right angle we use the following formulae:

Sine Rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
Cosine Rule	$a^2 = b^2 + c^2 - 2bc \cos A$
Area of a Triangle	$\text{Area} = \dfrac{1}{2}ab \sin C$

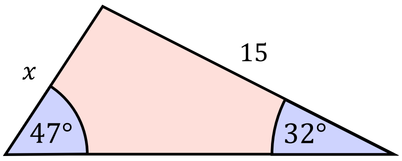
Much like a right angled triangle, we need to be able to label a non-right angled triangle correctly. Assuming the three lengths to be a, b and c – **in any particular order** – we need to ensure that angle A faces length a, angle B faces length b and angle C faces length c. This gives a triangle which **might** look like this:



Once we are satisfied we have labelled the triangle correctly, we can look to apply the rules above. As a guide, only use the formula for the area of a triangle when requested or if the area of the triangle is presented – usually this means you are being asked to find a component of the formula. I try to apply the Sine Rule to the scenario firstly, and if we don't have the applicable information to use this then revert to the Cosine Rule.

Unauthorised publication, distribution
or reproduction of these notes is prohibited.

Ex. 1 Find the value of x in the triangle below. Give your answer correct to two decimal places.



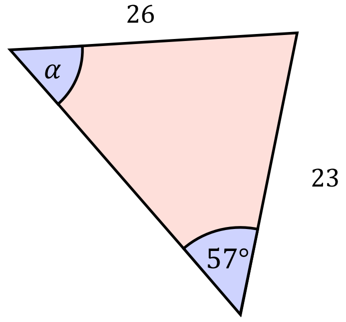
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 32°} = \frac{15}{\sin 47°}$$

$$x = \frac{15 \sin 32°}{\sin 47°}$$

$$x = 10 \cdot 87$$

Ex. 2 In the triangle below, find, correct to one decimal place, the value of α.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{23}{\sin \alpha} = \frac{26}{\sin 57°}$$

$$\frac{23 \sin 57°}{26} = \sin \alpha$$

$$\sin^{-1}\left(\frac{23 \sin 57°}{26}\right) = \alpha$$

$$47 \cdot 9° \approx \alpha$$

NB – Remember, when using the Sine Rule to calculate an angle we need to take the value the calculator gives us (in this case $\alpha_1 \approx 47 \cdot 9°$) and the value which is obtained when this is subtracted from 180°.

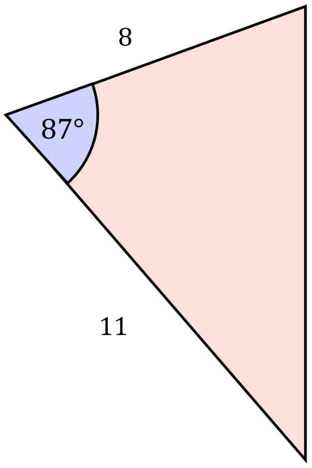
So, for the above example:

$\alpha_1 \approx 47 \cdot 9°$

$\alpha_2 \approx 180° - 47 \cdot 9° \approx 132 \cdot 1°$ - but this would need to be discarded as the angles would then add/sum to a value greater than 180°.

Unauthorised publication, distribution or reproduction of these notes is prohibited.

Ex. 3 Find the value of x, correct to the nearest integer, in the triangle below:



$a^2 = b^2 + c^2 - 2bc\cos A$

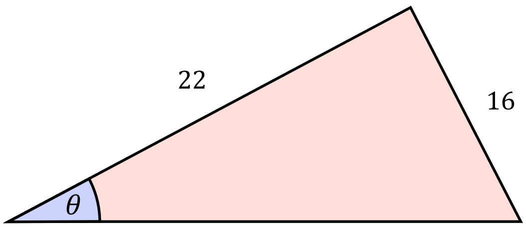
$x^2 = 8^2 + 11^2 - 2(8)(11)\cos 87°$

$x^2 = 175 \cdot 79$

$x \approx 13 \cdot 26$

$x \approx 13$

Ex. 4 Find the measure of angle θ, correct to two decimal places, in the triangle below:



$a^2 = b^2 + c^2 - 2bc\cos A$

$16^2 = 22^2 + 23^2 - 2(22)(23)\cos\theta$

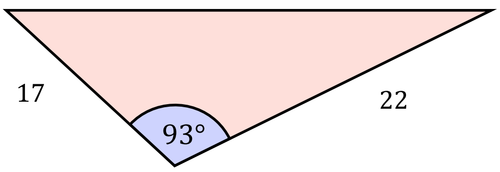
$\cos\theta = \dfrac{22^2 + 23^2 - 16^2}{2(22)(23)}$

$\theta = \cos^{-1}\left(\dfrac{22^2 + 23^2 - 16^2}{2(22)(23)}\right)$

$\theta \approx 41 \cdot 58°$

Unlike the Sine Rule, when we are using the Cosine Rule to calculate an angle there will only be one answer – as cos is negative in the second quadrant. We will further consider why this is the case in Trigonometry II.

Ex. 5 Find the area of the given triangle, correct to one decimal place.



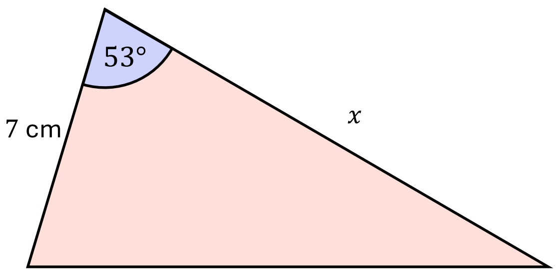
$\text{Area} = \dfrac{1}{2}ab\sin C$

$A = \dfrac{1}{2}(17)(22)\sin 93°$

$A \approx 186 \cdot 7$

Unauthorised publication, distribution or reproduction of these notes is prohibited.

Ex. 6 If the area of the triangle below is 25 cm², find the value of x correct to the nearest integer.



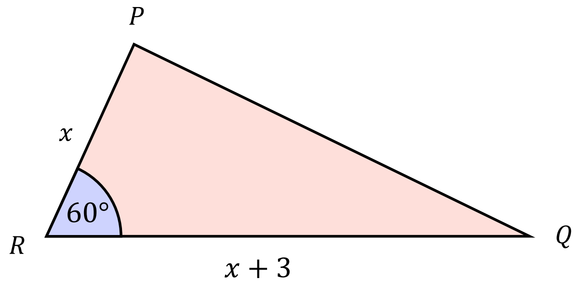
$$25 = \frac{1}{2}ab\sin C = \frac{1}{2}(7)(x)\sin 53°$$

$$50 = 7x\sin 53°$$

$$\frac{50}{7\sin 53°} = x$$

$$9 \approx x$$

Ex. 7 Given that the area of ΔPQR is $10\sqrt{3}$, find the value of x.



$$10\sqrt{3} = \frac{1}{2}ab\sin C = \frac{1}{2}(x)(x+3)\sin 60°$$

$$10\sqrt{3} = \frac{1}{2}(x)(x+3)\left(\frac{\sqrt{3}}{2}\right)$$

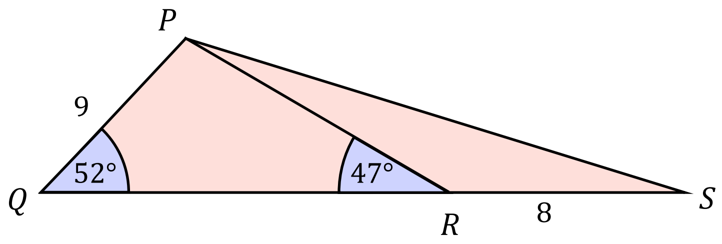
$$40 = x^2 + 3x$$

$$0 = x^2 + 3x - 40$$

$$0 = (x+8)(x-5)$$

$$-8 \neq x, \quad 5 = x$$

Ex. 8 Find the value of $|PR|$, correct to one decimal place, and the value of $|PS|$, correct to two decimal places:



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 52°} = \frac{9}{\sin 47°}$$

$$x = \frac{9\sin 52°}{\sin 47°}$$

$$|PR| \approx 9 \cdot 7$$

$$|\angle PRS| = 180° - 47° \quad \Longrightarrow |\angle PRS| = 133°$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = (9 \cdot 7)^2 + (8)^2 - 2(9 \cdot 7)(8)\cos 133°$$

$$a^2 = 263 \cdot 94$$

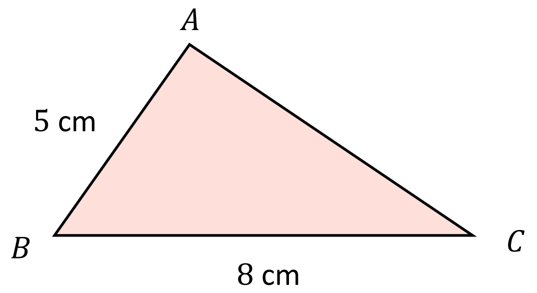
$$a \approx 16 \cdot 25$$

Unauthorised publication, distribution or reproduction of these notes is prohibited.

Ex. 9 The area of ΔABC is $16 \cdot 58$ cm². Find:

i. $|\angle ABC|$, correct to the nearest degree.

ii. $|AC|$, correct to one decimal place.



i.

$$A = \frac{1}{2}ab\sin C$$

$$16 \cdot 58 = \frac{1}{2}(5)(8)\sin C$$

$$\frac{16 \cdot 58}{20} = \sin C$$

$$\sin^{-1}\left(\frac{16 \cdot 58}{20}\right) = C$$

$$56° \approx C$$

ii.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = (5)^2 + (8)^2 - 2(5)(8)\cos 56°$$

$$a^2 = 44 \cdot 26$$

$$a \approx 6 \cdot 7 \text{ cm}$$

Unauthorised publication, distribution or reproduction of these notes is prohibited.